Introduction to probability with MATLAB, Spring 2014
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Exercise set 1

1. Consider the throw of one six-sided die. The set of possible outcomes is $\Omega = \{1, 2, 3, 4, 5, 6\}$. Write down three *different* events, each of which has probability $\frac{1}{2}$.

In problems 2–5 we shall assume an *equiprobable sample space*. This means that the sample space Ω contains a finite number n possible outcomes, of which exactly one will be true, and we assume each outcome to have the same probability $\frac{1}{n}$. This implies that if A is an event (that is, a set of possible outcomes, ie. a subset of Ω), and contains n elements, then

$$P(A) = \frac{\#A}{\#\Omega} = \frac{\#A}{n}.$$

Here # denotes the "number of elements" function for sets. The idea is to prove these elementary properties probability for an equiprobable sample space simply by counting the elements.

2. Prove that (for any event $A \subset \Omega$) it holds that $0 \leq P(A) \leq 1$. Note: $\subset$ denotes the "subset" relation, where equality is possible (ie. also $\Omega \subset \Omega$).

3. Prove that $P(A^c) = 1 - P(A)$, where A^c denotes the set complement (the set of elements in Ω which are not elements of A).

4. Prove that $P(A) \leq P(B)$ for any events A, B such that $A \subset B$.

Give an example of such two events in the tossing of one die, calculate their probabilities and verify that the claim holds.

5.

(a) Prove that for any two *disjoint* events A and B (that is, events whose intersection is empty) it holds that $P(A \cup B) = P(A) + P(B)$. Prove that for any two events A and B in general, it holds that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Give an example of two disjoint events in die-tossing and verify that the claim holds.

(c) Give an example of two non-disjoint events (that is, events whose intersection is not empty) in die-tossing, and verify that the claim holds.

6. A certain population contains 1000 persons, and one of them is chosen at random. We know that 400 of them are men (the rest are women), and 200 persons of the population are left-handed (the rest being right-handed).

Calculate the probabilities of the following events. If a probability is cannot be known precisely, explain why not, and prove lower and upper bounds as tight as possible.

(a) The person is a woman.

(b) The person is a left-handed man.

(c) The person is left-handed **or** a man (or both).

(d) The person is a right-handed woman.

7. A fair coin is tossed three times. List all possible outcomes, as sequences of three numbers with 0 indicating tails and 1 indicating tails. (There should be 8 different outcomes.)

We will assume that each of the 8 outcomes has the same probability. What is the probability of getting

(a) exactly 0 heads

(b) exactly 1 heads

(c) exactly 2 heads

(d) exactly 3 heads

(e) at most 2 heads

(f) less than 2 heads

(g) an even number of heads?

8. A wooden cube has its six faces painted. The cube is then sawed into three in each three directions (X, Y and Z), obtaining $3^3 = 27$ small cubes. The small cubes are put into a bag, and one small cube is picked at random. What is the probability that it has exactly k painted faces ($k = 0, 1, 2, 3$)?

9. Evaluate the integral

$$\int_0^5 e^{x+2} dx.$$

(We will need a little integral calculus later in the course, so it is useful to work out your calculus skills a little.)

10. Consider the set of possible die-tossing outcomes $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let $S = \{1, 2\}$ ("small" outcomes), $B = \{5, 6\}$ ("big" outcomes), $E = \{2, 4, 6\}$ (even outcomes), and $O = \{1, 3, 5\}$ (odd outcomes).

Let $k \in \Omega$. Which of the following 10 events are the same? Note that we are not asking which events have same *probability*, but which are the *same event*, that is, the *same set* of outcomes.

1. k is even **and** big
2. k is even **or** big
3. k is odd
4. k on even **but not** big
5. k is big **but not** even
6. $k \in E^c$
7. $k \in B \setminus E$
8. $k \in E \cap B$
9. $k \in E \cup B$
10. $k \in E \setminus B$

Write down the elements of each set listed above.

Note: In mathematics, "or" is understood as inclusive; that is, "A or B" is true if either one or both of A and B are true.