

INTRODUCTION TO BIFURCATION THEORY PROJECT: LORENZ SYSTEM

The system of Lorenz (1963)

$$\begin{aligned}\dot{x} &= \sigma(y - x) \\ \dot{y} &= \mu x + x - y - xz \\ \dot{z} &= -\beta z + xy\end{aligned}$$

is a simplified model for atmospheric convection. Notice, that in the classical version of this model $\mu = r - 1$ (see Lecture notes). Constants σ, β are assumed positive and fixed!

Bit of history: The original model was a partial differential system studied by Rayleigh (1916). Saltzman (1962) modified this model by taking Fourier transformations and restricted it to finite dimensions, proposed by Lorenz (1960). In Lorenz (1963) it was formulated as above (with $\mu = r - 1$) and analyzed. σ is the Prandtl number, x is proportional to the intensity of the convection motion and y is proportional to the temperature difference between ascending and descending currents. The variable z is proportional to the distortion of the vertical temperature profile from linearity. The above equations give fairly realistic results when the Rayleigh number is slightly supercritical, but only when weak convection occurs.

The task: Show, that at the origin $(x, y, z) = (0, 0, 0)$ the system undergoes a pitchfork bifurcation when μ passes zero. You must (i) transform the system into a standard form (ii) find the center manifold (iii) restrict the vector field to this center manifold and do the necessary analysis. Furthermore, (iv) write a software program that does these calculations.

Instructions and stuff:

Instructions on writing the report: Report must clearly (!) indicate the calculations. It can be written by hand or with a software (word, latex, maple). You can either leave this report in my "postbox", or, you may scan it (if needed) and send it by mail. The maple program must be sent by mail. You may, if you want, to do it with someone together! Maximum two people per one report.

Points: You will receive points for all the above steps (i)-(iv), approx. 1.5 points per step.

Deadline: The project must be returned by Sunday 22.12.2013. IF you need more time, you must agree this with me.

I propose that you start working on the project before the exam, as I believe it will be a good preparation for it!

Please ask for further instructions!