INTRODUCTION TO BIFURCATION THEORY

Exercises 21-11-2013

Consider

$$\dot{x} = -x + y^2$$
$$\dot{y} = -2x^2 + 2xy^2$$

31. (8 points) Find all the equilibria. As one equilibrium is the origin, compute the invariant manifolds belonging to this equilibrium.

32. (16 points) Another equilibrium is $(\hat{x}, \hat{y}) = (1, 1)$.
(a) compute the invariant manifolds connected to this equilibrium by shifting the equilibrium to the origin. <u>Don't</u> transform it to the simple block diagonal form (i.e. do not apply map T). Hint: the first-order terms of the manifolds are not zero, that is, $Dh(\cdot) \neq 0$.
(b) compute the invariant manifolds connected to this equilibrium by shifting it to the origin AND applying map T.

33. (8 points) Draw as complete phase-portrait as you can. You need to (i) investigate the stability of all the invariant manifolds you just calculated. That is, you need to restrict the vector field to those manifolds (ii) be careful, this is a very special system! Why?

34. (10 points) Work out a Maple program that does all these above calculations. Hint: Use the command mtaylor for taylor expansions.

Please ask for further instructions if needed!