INTRODUCTION TO BIFURCATION THEORY

Exercises 14-11-2013 (This exercises session will be held jointly with the previous one 7-11-2013!)

27. (4 points) Show that $\phi(t, X) = e^{At}X$, where A is an $n \times n$ matrix, is a smooth dynamical system on $\mathbb{R}^n$.

28. (4 points) Show that all polynomials are entire functions.

29. (6 points) Consider

$$\dot{X} = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix} X.$$

Show that whether the vector field at the unit circle points inside or outside of the unit circle depends completely on the real part of the eigenvalues. Hint: change the system to polar coordinates.

30. (6 points) Consider the SIR-model with vaccinations

$$\dot{S} = (1-p)m - (\beta I + m)S$$
$$\dot{I} = \beta IS - (g+m)I$$

(See Lecture notes). Determine the stability of the equilibria.