INTRODUCTION TO BIFURCATION THEORY

Exercises 10-10-2013

18. (4 points) Consider a system $\dot{X} = AX$ where A has repeated eigenvalues and more than one linearly independent eigenvector. Find a linear map T that transform this system to $\dot{T} = BY$ where B is in canonical form

$$B = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

19. (12 points) For each of the following systems of the form $\dot{X} = AX$
(a) Find the eigenvalues and eigenvectors of A.
(b) Find the matrix T that puts A in canonical form ($B = T^{-1}AT$).
(c) Find the general solution of $\dot{X} = AX$ by (i) deriving it using the eigenvalues and eigenvectors (if you know how to) (ii) finding the general solution to $\dot{Y} = BY$ and using the map T
(d) sketch the phase portraits of $\dot{X} = AX$ and $\dot{Y} = BY$.

$$(I) \quad A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (II) \quad A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} \qquad (III) \quad A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$

20. (12 points. Note that this is an extended version of exercise 17) Damped harmonic oscillator (DHO) satisfies the following equation

$$\ddot{x} + 2\zeta w_0 \dot{x} + w_0^2 x = 0,$$

where $w_0 > 0$ is the undamped angular frequency of the oscillator and $\zeta \geq 0$ is the damping ratio.
(a) Set up a planar linear system for DHO and find the eigenvalues
(b) For each type of eigenvalues find the general solution of this system by (i) deriving it using the eigenvalues and eigenvectors (if you know how to) (ii) finding the general solution to $\dot{Y} = BY$ where B is the canonical form of the system and then using a linear transformation.
(c) Draw the phase portrait for each of these cases. Can you see how the phase portraits transform from one to the other?
(d) Draw how the position of the mass moves with time, i.e. make a $(t, x(t))$-plot.