INTRODUCTION TO BIFURCATION THEORY

Exercises 3-10-2013

12. (4 points) Consider a planar system $\dot{X} = AX$ with

$$A = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

and $\lambda_1 < \lambda_2$. Find the general solution and draw the phase portrait for the special case where one of the eigenvalues is 0.

13. (4 points) Consider the 'Example (center)' found in the lecture notes under the section 'Complex Eigenvalues'. In this example we looked at a special matrix

$$A = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix}$$

for which we found two eigenvalues $\lambda_{1,2} = \pm i\beta$, and using the eigenvalue $i\beta$ we derived the general solution. Show that it doesn't matter which one we would have chosen, i.e. show that using the eigenvalue $-i\beta$ we get the same general solution.

14. (4 points) Consider the system

$$A = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

with $\lambda \neq 0$. Show that all solutions tend to or away from the origin tangentially to the eigenvector $(1, 0)$.

15. (4 points) Find the general solution and describe completely the phase portrait for

$$\dot{X} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} X.$$

16. (4 points) Prove that

$$\alpha e^{\lambda t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta e^{\lambda t} \begin{pmatrix} t \\ 1 \end{pmatrix}$$

is the general solution of

$$\dot{X} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} X.$$

17. (8 points) Damped harmonic oscillator (DHO) satisfies the following equation

$$\ddot{x} + 2\zeta w_0 \dot{x} + w_0^2 x = 0,$$

where $w_0 > 0$ is the undamped angular frequency of the oscillator and $\zeta > 0$ is the damping ratio. (a) Set up a planar linear system for DHO and find the eigenvalues (b) Find the general solution of this system (for all the different types of eigenvalues, if you know how to) (c) Draw the phase portrait for each of these cases. Can you see how the phase portraits transform from one to the other? (d) Draw how the position of the mass moves with time, i.e. make a $(t, x(t))$-plot.