INTRODUCTION TO BIFURCATION THEORY

Exercises 26-9-2013

6. (4 points) Show that if V_0 is an eigenvector for A with eigenvalue λ, then any nonzero scalar multiple of V_0 is also an eigenvector for A with eigenvalue λ.

7. (4 points) Prove the following proposition:

The planar system $\dot{X} = AX$ has

1. A unique equilibrium point $(0, 0)$ if $\det A \neq 0$.
2. A straight line of equilibrium points if $\det A = 0$ (and A is not the 0 matrix)

8. (4 points) Prove that the list of vectors $(V_1, \ldots, V_m)$ is linearly independent if and only if every vector in some vector space F can be uniquely written as a linear combination of $(V_1, \ldots, V_m)$.

9. (4 points) Show that if we have two distinct real eigenvalues λ_1 and λ_2 with eigenvectors V_1 and V_2, then V_1 and V_2 are linearly independent.

10. (4 points) Prove the following theorem:

Let

$$\dot{X} = AX$$

be a planar linear system. Suppose that $Y_1(t)$ and $Y_2(t)$ are solutions of this system, and that vectors $Y_1(0)$ and $Y_2(0)$ are linearly independent. Then

$$X(t) = \alpha Y_1(t) + \beta Y_1(t)$$

is the unique solution of this system that satisfies $X(0) = \alpha Y_1(0) + \beta Y_1(0)$.

11. (6 points) (Computer exercise) Damped harmonic oscillator satisfies the following equation

$$\ddot{x} + 2\zeta w_0 \dot{x} + w_0^2 x = 0,$$

where w_0 is the undamped angular frequency of the oscillator and ζ is the damping ratio. (a) Solve (by using for example Maple) the equation with initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$. (b) plot the solution (make a $(t, x(t))$-plot) for $w_0 = 5$

and for $\zeta = 0, 0.2, 1$ and 1.3. Display all four plots in one figure. Use different colors for different plots.