INTRODUCTION TO BIFURCATION THEORY

Exercises 5-12-2013

37. (8 points) Consider

$$\begin{aligned}\dot{x} &= -y + xz\\ \dot{y} &= x + yz\\ \dot{z} &= -z - x^2 - y^2 + z^2\end{aligned}$$

Determine the stability of the equilibrium at the origin. Hint: after restricting the vector field on the center manifold it will be helpful to use polar coordinates and the identity $r\dot{r} = x\dot{x} + y\dot{y}$, where $x = r\cos\theta, y = r\sin\theta$.

38. (a) (3 points) Consider the saddle-node bifurcation. For the case $(\frac{\partial^2 f}{\partial x^2}(0,0)/\frac{\partial f}{\partial \mu}(0,0)) > 0$, under which conditions the upper part of the curve of equilibria is stable and the lower unstable?
(b) (3 points) Consider the transcritical bifurcation. For the case $(\frac{\partial^2 f}{\partial x^2}(0,0)/\frac{\partial^2 f}{\partial x \partial \mu}(0,0)) > 0$, under which conditions the curve $x = 0$ is stable for $\mu > 0$ and the other curve of equilibria unstable?

39. (6 points) Consider $\dot{x} = f(x,\mu)$, $\mu, x \in \mathbb{R}$. Give conditions under which the system undergoes a pitchfork bifurcation. Hint: Use procedures used in lecture notes when deriving conditions for the transcritical and saddle-node bfurcations.

40. (6 points) Consider

$$\begin{aligned}\dot{x} &= \mu + x^2 + y^2\\ \dot{y} &= -y + x^2.\end{aligned}$$

What bifurcation this system undergoes?