INTRODUCTION TO BIFURCATION THEORY

Exercises 19-9-2013

1. (4 points) Show that $x(t) = Ke^{\mu t}$ is the only solution to

$$\dot{x} = \mu x, \quad x \in \mathbb{R} \tag{1}$$

with $x(0) = K$.
Hint: Letting $u(t)$ to be any solution, compute the derivative of $u(t)e^{-\mu t}$.

2. (4 points) Above (Example 1 in the Lecture notes) is a very simplistic model of population growth, for example, the assumption of growth without bound is naive. The following *logistic population growth model* is bit more realistic

$$\dot{x} = \mu x(1 - \frac{x}{N}), \quad x \in \mathbb{R},$$

where $\mu > 0$ is the initial growth rate and N is the sort of "ideal" population or *carrying capacity* (why is it called carrying capacity?). Find and analyze the general solution.

3. (4 points) Draw the direction of the flow (by calculating the nullclines and finding which direction the flow points at those nullclines) for the SIR-model with vaccinations when $p < p_c$ (see Lecture notes). Are there any difficulties to do this?

4. (4 points) (Ok, I said it won't be an exercise, sorry) (a) Write up a procedure with a system of Example 3 (see Lecture notes) which executes the plot when you call the procedure. (b) Give it two initial conditions, one close to the origin and one sufficiently far away from the origin. (c) Set initial conditions and μ to be the input parameters, and make all the variables inside the procedure local such that when changing parameter values outside the procedure it doesn't affect the results! (d) Try your procedure for different input values.

5. (4 points) Plot the chaotic attractor from Example 4 (Lecture notes). Use command DEplot3d and two initial conditions: $x_1(0) = 3, x_2(0) = 15, x_3(0) = 1$ and $x_1(0) = 4, x_2(0) = -2, x_3(0) = 1$. Draw the two solution curves with different colours. Hint: Use two DEplot3d commands one for each initial condition and then display the plots.