X-ray tomography

砂夢理

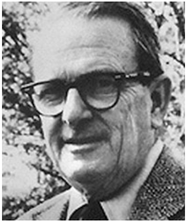
Samuli Siltanen

Department of Mathematics and Statistics
University of Helsinki, Finland

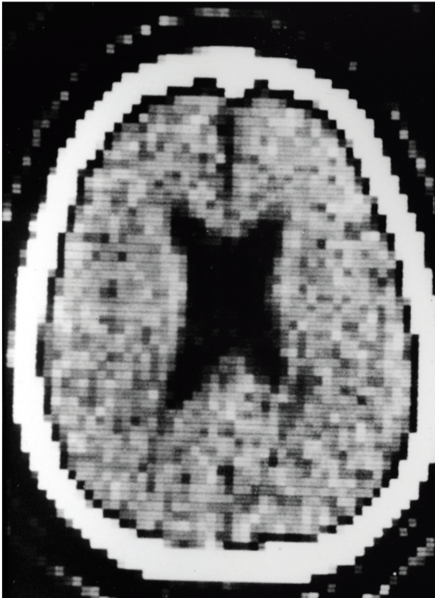
汁太念

November 6, 2012

Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography



Hounsfield (top) and Cormack received Nobel prizes in 1979.

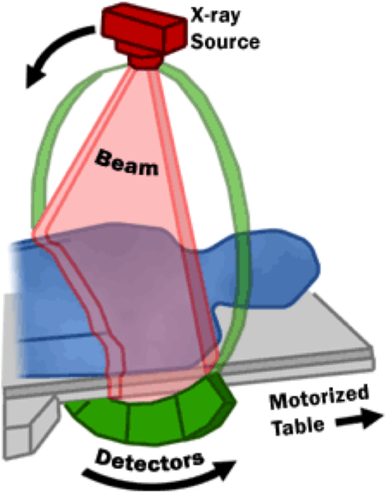


Reconstruction of a function from its line integrals was first invented by Johann Radon in 1917

Johann Radon (1887-1956)

$$f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\overline{F_p}(q)}{q}$$

Traditional X-ray tomography requires many projection images using small angular steps



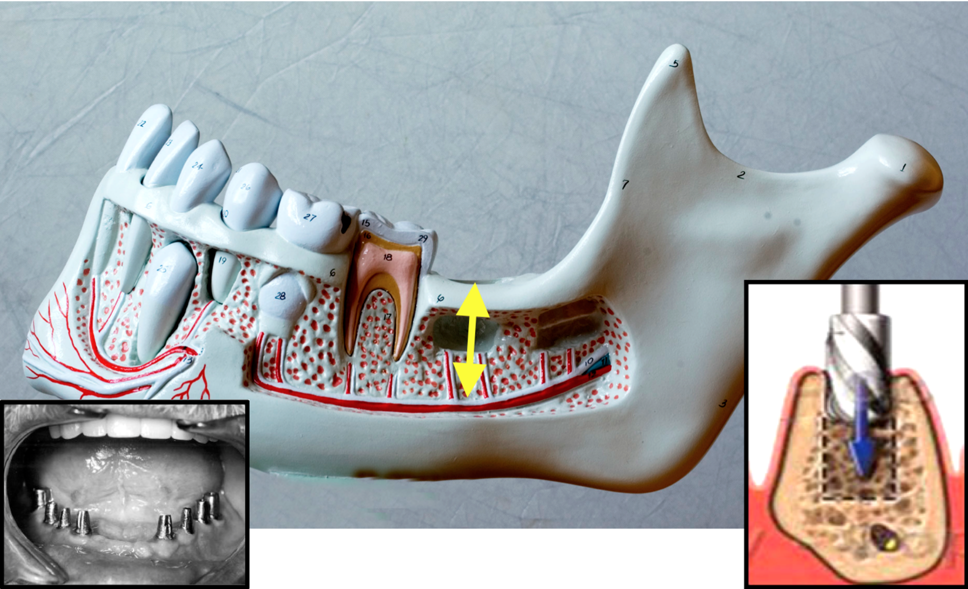
$$f(x) = \frac{1}{4\pi^2}\int_{S^1}\int_{\mathbb{R}} \frac{\frac{d}{ds}(Rf)(\theta, s)}{x\cdot\theta - s}\, ds\, d\theta$$

Due to the radiation dose, a CT scan is only appropriate for seriously ill patients

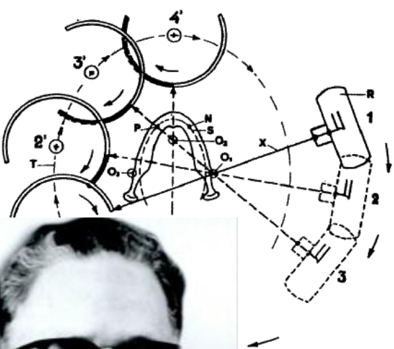
In filtered backprojection, the mathematical reconstruction formula assumes dense angular sampling of full-angle data. The inverse problem is only mildly ill-posed, but
The chosen mathematics requires high radiation dose.

Think the opposite: take as few X-ray images as possible. The inverse problem is very ill-posed. Use advanced mathematics to form a reconstruction that is good enough for the clinical task.
The low level of radiation dose requires new mathematics and more computational power.

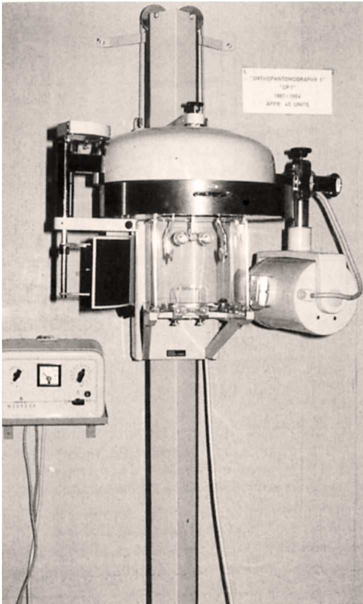
Application: dental implant planning, where a missing tooth is replaced with an implant



Panoramic dental imaging shows all the teeth simultaneously



Panoramic imaging was invented by Yrjö Veli Paatero in the 1950's.



Nowadays, a digital panoramic imaging device is standard equipment at dental clinics

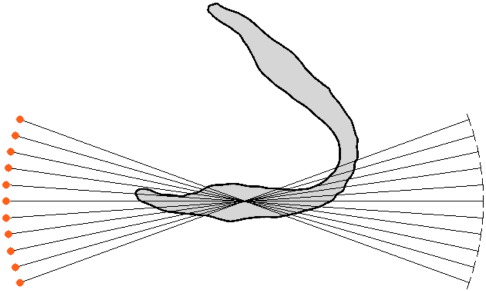
Panoramic images are not good enough for dental implant planning because of geometric distortion.

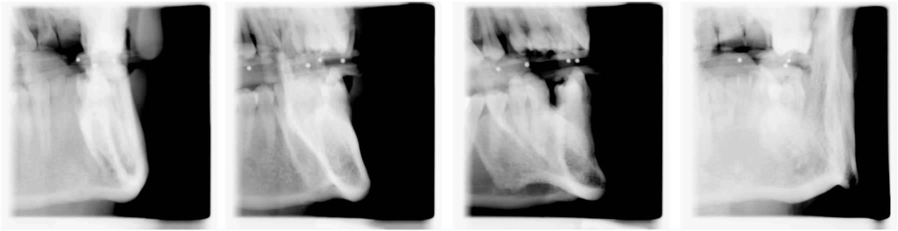
We reprogram the panoramic X-ray device so that it collects projection data by scanning

11 projection images of the mandibular area

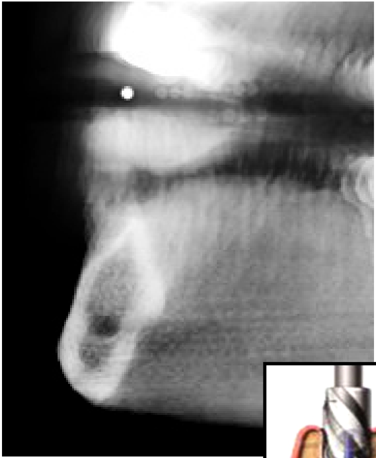
40 degrees angle of view

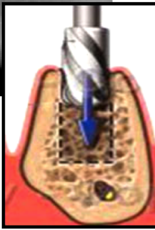
1000×1000 image size, formed by a scanning movement





Here are example images of a patient





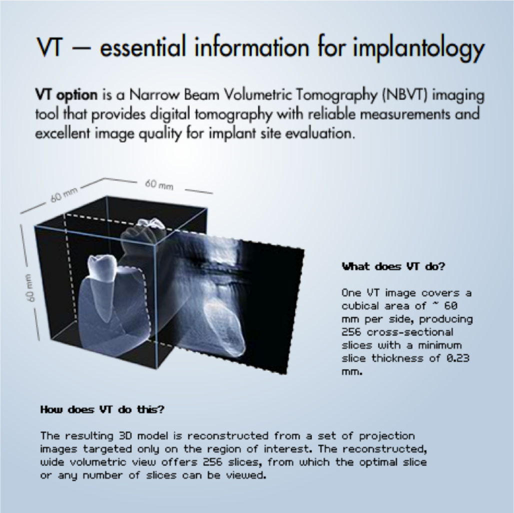
Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke **2006**,
Kolehmainen, Lassas & S **2008**, Cederlund, Kalke & Welander **2009**,
Hyvönen, Kalke, Lassas, Setälä & S **2010**,
United States patent 7269241

This low-dose 3D imaging technique has been commercialized by Palodex Group

The VT device has been in the market from year 2007.

Remarkably, an existing 2D panoramic imaging device becomes a 3D imaging product just by a software update.

The core of that update is an inversion algorithm.

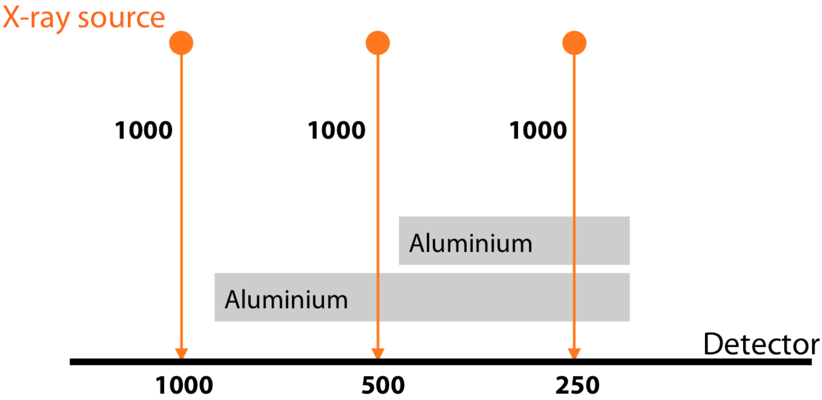


The radiation dose of the VT device is the lowest among 3D dental imaging modalities

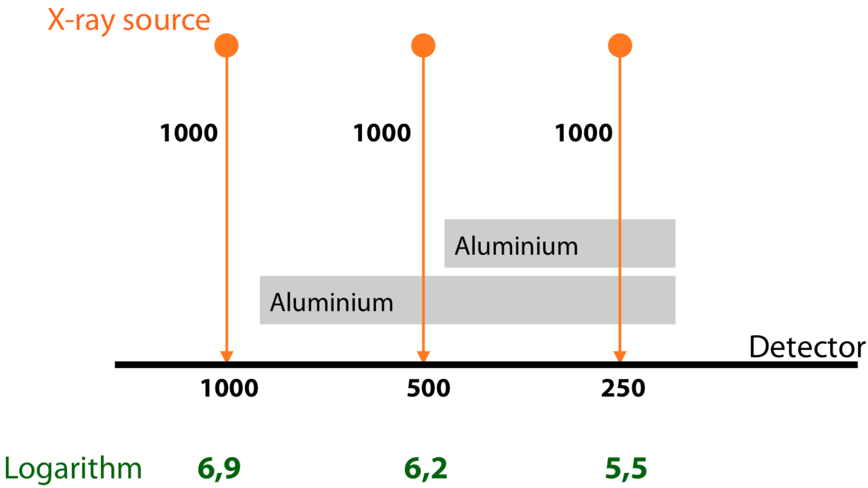
Modality	μ**Sv**
Head CT	2100
CB Mercuray	558
i-Cat	193
NewTom 3G	59
VT device	**13**

Ludlow, Davies-Ludlow, Brooks & Howerton **2006**

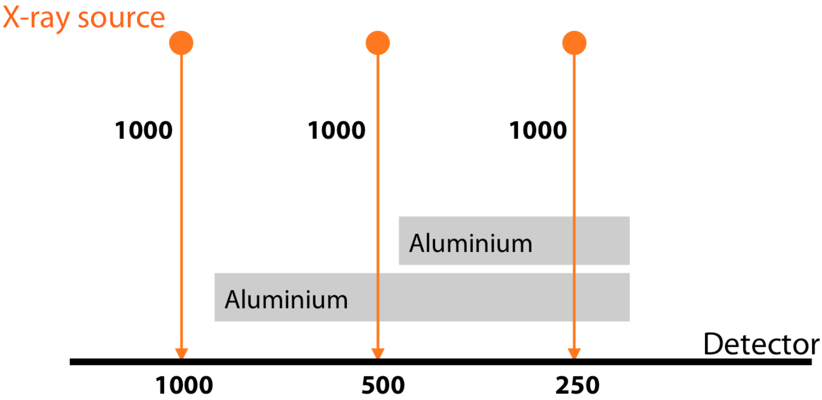
Mathematical interpretation of X-ray measurements



Mathematical interpretation of X-ray measurements



Mathematical interpretation of X-ray measurements

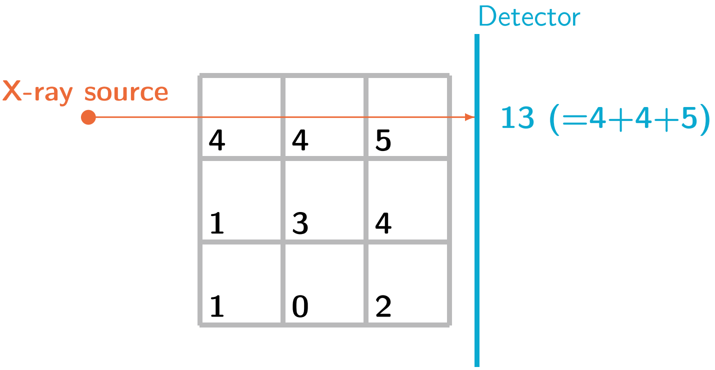


Logarithm	6,9	6,2	5,5
Density	0,0	0,7	1,4

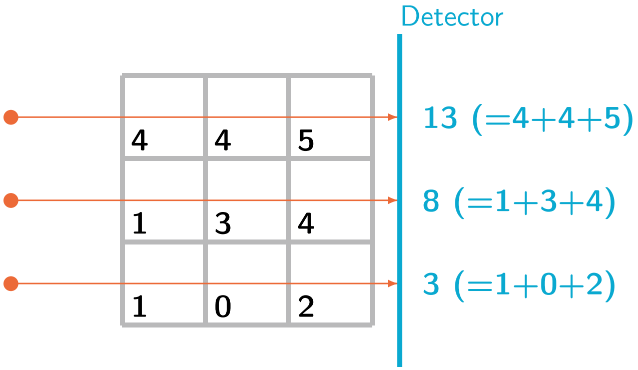
Let us study a simple two-dimensional example of tomographic imaging

4	4	5
1	3	4
1	0	2

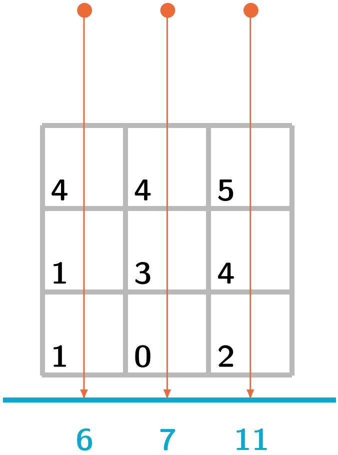
Tomography is based on measuring densities of matter using X-ray attenuation data



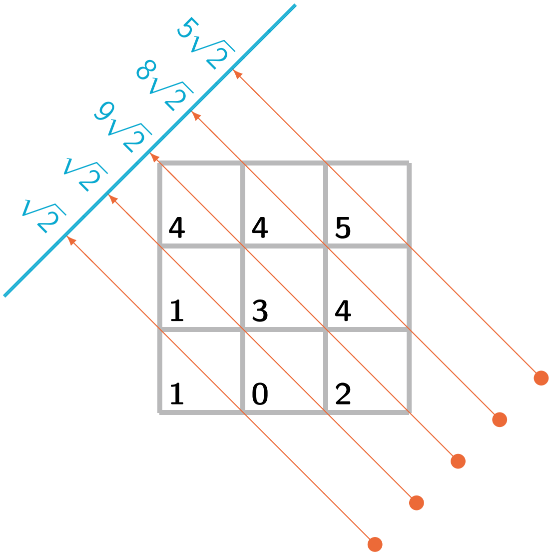
A projection image is produced by parallel X-rays and several detector pixels (here three pixels)



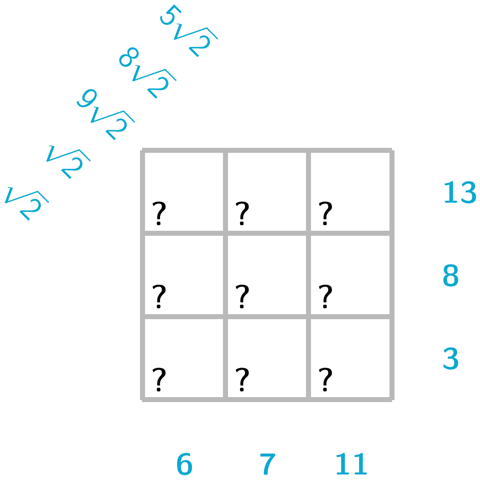
For tomographic imaging it is essential to record projection images from different directions



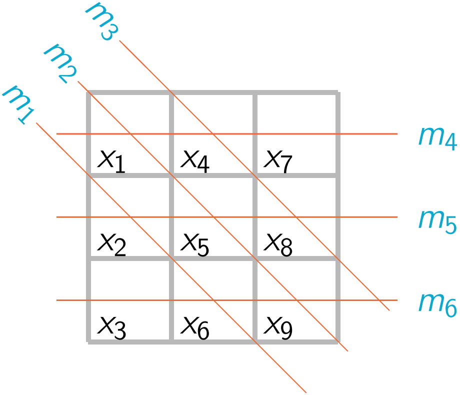
The length of X-rays traveling inside each pixel is important, thus here the square roots



The inverse problem of tomography is to reconstruct the interior from X-ray data



We write the reconstruction problem in matrix form and assume Gaussian noise



$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}, \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix},$$

$$m = Ax$$

Our measurement model is $m = Ax + \varepsilon$ with independently distributed Gaussian noise (white noise) with standard deviation $\sigma > 0$.

This is the matrix equation $Ax = m$ related to the measurement on the previous slide

$$\begin{bmatrix} 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix}.$$