Exercise 7

1. Let the notation be as in the lecture notes.

Recall, that $p_i = \sum_j P_{ij}$ and $q_j = \sum_i P_{ij}$.

Then
$$p_i' = \sum_j P_{ij}' = \sum_j [(1-r)P_{ij} + r p_i q_j]$$
$$= (1-r)\sum_j P_{ij} + r p_i \sum_j q_j$$
$$= (1-r)p_i + r p_i$$
$$= p_i \quad , \text{ for all } i$$

Similarly, we can show that $q_j' = q_j$ for all j.

2. By definition, we have
$$p_1 = \sum_{j=1}^{2} P_{1j} = P_{11} + P_{12} \overset{\text{def}}{=} x_1 + x_2$$
$$p_2 = P_{21} + P_{22} = x_3 + x_4$$
$$q_1 = P_{11} + P_{21} = x_1 + x_3$$
$$q_2 = P_{12} + P_{22} = x_2 + x_4$$

Then,
$$D_{11} = P_{11} - p_1 q_1 = x_1 - (x_1 + x_2)(x_1 + x_3)$$
$$= x_1(1 - x_1 - x_2 - x_3) - x_2 x_3$$
$$= x_1 x_4 - x_2 x_3 \overset{\text{def}}{=} D$$

$$D_{12} = P_{12} - p_1 q_2 = x_2 - (x_1 + x_2)(x_2 + x_4)$$
$$= x_2(1 - x_1 - x_2 - x_4) - x_1 x_4$$
$$= x_2 x_3 - x_1 x_4 = -D$$

Similarly, we can calculate $D_{21} = -D$ and $D_{22} = D$.

3. Recall, that

$$\overline{w} = \sum_{i,j} w_{ij} x_i x_j$$
$$= \overbrace{(a_{11} + b_{11})}^{w_{11}} x_1 x_1 + 2 \overbrace{(a_{11} + b_{12})}^{w_{12} = w_{21}} x_1 x_2 +$$
$$+ \underbrace{(a_{11} + b_{22})}_{w_{22}} x_2 x_2 + \ldots$$
$$= a_{11}(x_1^2 + 2 x_1 x_2 + x_2^2) + \ldots$$
$$= a_{11}(x_1 + x_2)^2 + \ldots$$
$$= a_{11} p_1^2 + \ldots$$

$\Rightarrow \quad \overline{w} = \overline{a} + \overline{b}$, where

$$\overline{a} = a_{11} p_1^2 + 2 a_{12} p_1 p_2 + a_{22} p_2^2$$
$$\overline{b} = b_{11} q_1^2 + 2 b_{12} q_1 q_2 + b_{22} q_2^2$$

are the single locus mean fitnesses.

$\overline{w}$ is <u>only</u> a function of a's b's and the allele frequencies.