Exercises 3

- $P^*_{kl} Q_{ij,kl}$ prob. that a female A_iA_j mates with a male A_kA_l.

- $F_{ij,kl}$ - exp. # of offspring produced by A_iA_j & A_kA_l

- $R_{ij,kl\to mn}$ - (Mendelian) prob. that ij & kl produce an offspring A_mA_n

$P^*_{kl} Q_{ij,kl} F_{ij,kl}$ - exp # offspring of a female A_iA_j produced with a male A_kA_l

(b) - The expected # of offspring of a female A_iA_j

$$\sum_{k,l} P^*_{kl} Q_{ij,kl} F_{ij,kl}$$

(a) - The exp # of offspring of type A_mA_n of a female A_iA_j produced with a male A_kA_l

$$P^*_{kl} Q_{ij,kl} F_{ij,kl} R_{ij,kl\to mn}$$

(c) - The exp. # of offspring of type A_mA_n produced by female A_iA_j

$$\sum_{k,l} P^*_{kl} Q_{ij,kl} R_{ij,kl\to mn} F_{ij,kl}$$

(d) $$2 P'_{mn} = \frac{1}{\bar{Q}} \sum_{i,j} \sum_{k,l} P^*_{ij} P^*_{kl} Q_{ij,kl} F_{ij,kl} R_{ij,kl\to mn}$$

only if $m \neq n$

freq. of mn in the next generation

$$\bar{Q} = \sum_{i,j,k,l} P^*_{ij} P^*_{kl} Q_{ij,kl} F_{ij,kl}$$ (normalization factor)

mean mating success

Consider two alleles A_1, A_2, genotype freq are

$$P'_{mm} = \left(\sum_{i,j,k,l} P^*_{ij} P^*_{kl} Q_{ij,kl} R_{ij,kl \to mm} F_{ij,kl}\right) \frac{1}{\bar{Q}}$$

$m \neq n$ $$2P'_{mn} = \left(\sum P^*_{ij} P^*_{kl} Q_{ij,kl} R_{ij,kl \to mn} F_{ij,kl}\right) \frac{1}{\bar{Q}}$$

(e) $$P'_{11} = \frac{1}{\bar{Q}} \Big[P^*_{11} P^*_{11} Q_{11,11} F_{11,11} \cdot 1 + \tfrac{1}{2} P^*_{11} P^*_{12} Q_{11,12} F_{11,12} + \tfrac{1}{2} P^*_{11} P^*_{12} Q_{11,12} F_{11,12} + \tfrac{1}{2} P^*_{12} P^*_{11} Q_{12,11} F_{12,11} + \tfrac{1}{2} P^*_{12} P^*_{11} Q_{12,11} F_{12,11} + \tfrac{1}{4}\left[P^*_{12} P^*_{12} Q_{12,12} F_{12,12} \cdot 4\right] \Big]$$

$$= \frac{1}{\bar{Q}} \Big[P^*_{11} P^*_{11} Q_{11,11} F_{11,11} + P^*_{11} P^*_{12} Q_{11,12} F_{11,12} + P^*_{12} P^*_{11} Q_{12,11} F_{12,11} + P^*_{12} P^*_{12} Q_{12,12} F_{12,12} \Big]$$

Similarly for P'_{12} & P'_{22}

(f) Suppose $Q = Q_{g,h}$ & $F = F_{g,h}$ $\forall g,h$.

$$\bar{Q} = QF\left(P^*_{11}(P^*_{11} + 2P^*_{12} + P^*_{22}) + 2P^*_{12}(P^*_{11} + 2P^*_{12} + P^*_{22}) + P^*_{22}(P^*_{11} + 2P^*_{12} + P^*_{22})\right) = QF$$

$$\Rightarrow P'_{11} = P^*_{11}P^*_{11} + P^*_{11}P^*_{12} + P^*_{12}P^*_{11} + P^*_{12}P^*_{12} = P^*_{11}\overbrace{(P^*_{11} + P^*_{12})}^{P^*_1} + P^*_{12}\overbrace{(P^*_{11} + P^*_{12})}^{P^*_1}$$
$$= P^*_1 \cdot P^*_1 = (P^*_1)^2$$

Lets do the same thing for $2P'_{12}$

$$2P'_{12} = 2P^*_{11}P^*_{12} \cdot \tfrac{1}{2} + P^*_{11}P^*_{22} + P^*_{22}P^*_{11} + 2P^*_{22}P^*_{12} \cdot \tfrac{1}{2} + 2P^*_{12}P^*_{11}\tfrac{1}{2} + 2P^*_{12}2P^*_{12}\tfrac{1}{2} + 2P^*_{12}P^*_{22}\tfrac{1}{2}$$

$$= 2P^*_{11}P^*_{22} + 2P^*_{11}P^*_{12} + 2P^*_{12}P^*_{22} + 2P^*_{12}P^*_{12}$$

$$= 2\left(P^*_{11}\overbrace{(P^*_{12} + P^*_{22})}^{P^*_2} + P^*_{12}\overbrace{(P^*_{12} + P^*_{22})}^{P^*_2}\right)$$

$$= 2(P^*_2 P^*_1)$$

$$\Rightarrow \quad p'_1 = p'_{11} + p'_{12} = p^*_1 p^*_1 + p^*_1 p^*_2 = p^*_1 = p_1 \frac{V_1}{V}$$