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Mathematical theory of population genetics

Exercises 9.

1. Calculate E$[T]$, when

   (a) (3 points) T is exponentially distributed continuous random variable with rate parameter λ (i.e. λ is the rate at which events occur).

   (b) (3 points) T is geometrical distributed discrete random variable with parameter p (i.e. p is the probability of an event happening at times $t = 1, 2, 3, \ldots$).

2. (3 points) Check that in the Wright-Fisher model

$$G_{i,j} = \mathcal{O}(\frac{1}{N^2}) \quad \text{with } j < i - 1. \qquad (1)$$

   That is, the expression $G_{i,j}$, with $j < i - 1$, has all the terms of order $\frac{1}{N^2}$ or lower.

3. (8 points) Show that in the Moran model (as defined in the lecture notes) the probability that in one time-step a common ancestor event happens among the i sample lineages is

$$G_{i,i-1} = \left(1 - \frac{1}{N}\right) \frac{i(i-1)}{N(N-1)} = \binom{i}{2} \frac{2}{N^2}, \qquad (2)$$

   where N is the total population size, and $i \leq N$.

   Hint: A common ancestor event in the Moran model is taken to be as (backward in time) when a parent and its offspring coexist after reproduction among the i lineages. It is helpful to first calculate the probability that offspring and parent coexist among the i lineages given that the individual chosen for reproduction and death is not the same (which is a necessary condition for the parent and offspring to coexist in the population). You can think of it as tossing i balls without replacement into N boxes.