Tehtävä 2.a)

$$\left(\frac{\partial^2}{\partial t^2} - \Delta_x\right) U(x,t) = F(x,t)$$

toi tiedetään, siis $U(x,t)$ on aaltoyhtälön ratkaisu $\forall x, \forall t$

$$\left(\frac{\partial^2}{\partial t^2} - \Delta_x\right)\left[U(x,-t)\right] = (-1)^2 \frac{\partial^2}{\partial t^2} U(x,-t) - \Delta_x U(x,-t)$$

siis $$\left(\frac{\partial^2}{\partial t^2} - \Delta_x\right)\left[U(x,-t)\right] = \left(\frac{\partial^2}{\partial t^2} - \Delta_x\right) U\Big|_{(x,-t)} = F\Big|_{(x,-t)}$$

t.s. $$\left(\frac{\partial^2}{\partial t^2} - \Delta_x\right) U(x,-t) = F(x,-t)$$

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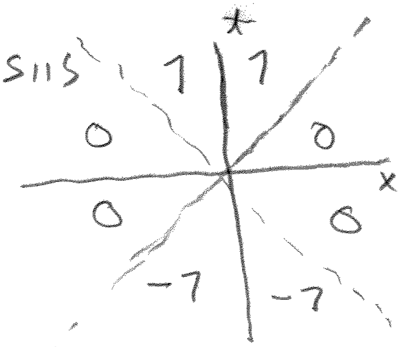
b)

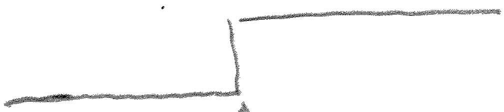
$$U(x,t) = \begin{cases} 1, & t > |x| \\ -1, & t < -|x| \\ 0, & \text{muulloin} \end{cases}$$

siis

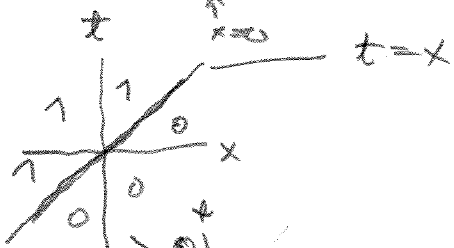


Kun Heaviside funktio (porrasfunktio)

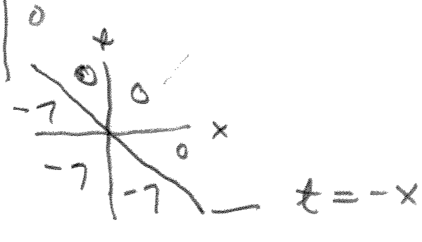
$$H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



$$F = H(t-x) = \begin{cases} 1, & t-x \geq 0 \\ 0, & \text{muuten} \end{cases}$$



$$G = -H(-x-t) = \begin{cases} -1, & -x-t \geq 0 \\ 0, & \text{muuten} \end{cases}$$



$$U(x,t) = F(x,t) + G(x,t) = \underline{H(t-x) - H(-x-t)}$$

(tai vaihtoehtoisesti
$$U(x,t) = \underline{H(x+t) - H(x-t)}\ )$$