HARJOITUS 1. 21.9.2012

1) a) $\dfrac{e^{i\theta}+e^{-i\theta}}{2} = \dfrac{\cos\theta + i\sin\theta + \cos(-\theta) + i\sin(-\theta)}{2} = \dfrac{2\cos\theta}{2}$

siis $\underline{\underline{\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}}}$

1 b) $\dfrac{e^{i\theta}-e^{-i\theta}}{2i} = \dfrac{\cos\theta + i\sin\theta - \cos(-\theta) - i\sin(-\theta)}{2i} = \dfrac{2i\sin\theta}{2i}$

joten $\underline{\underline{\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}}}$

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2) $z_1 = a_1 + ib_1$

$z_2 = a_2 + ib_2$

$* \Rightarrow z_1 = |z_1| e^{i\theta_1}$

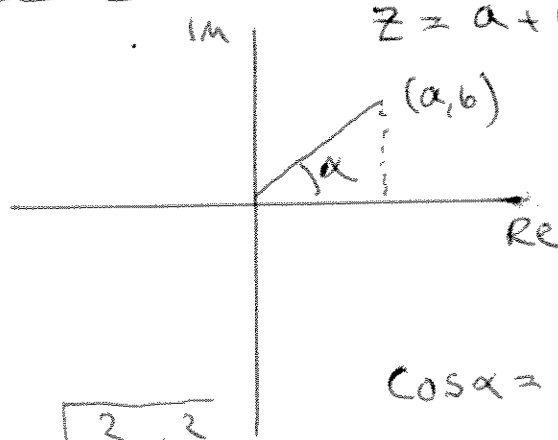
$z_2 = |z_2| e^{i\theta_2}$

$z_1 z_2 = |z_1||z_2| e^{i\theta_1} e^{i\theta_2}$

$z_1 z_2 = |z_1 z_2| e^{i(\theta_1+\theta_2)}$

*

$z = a + ib$



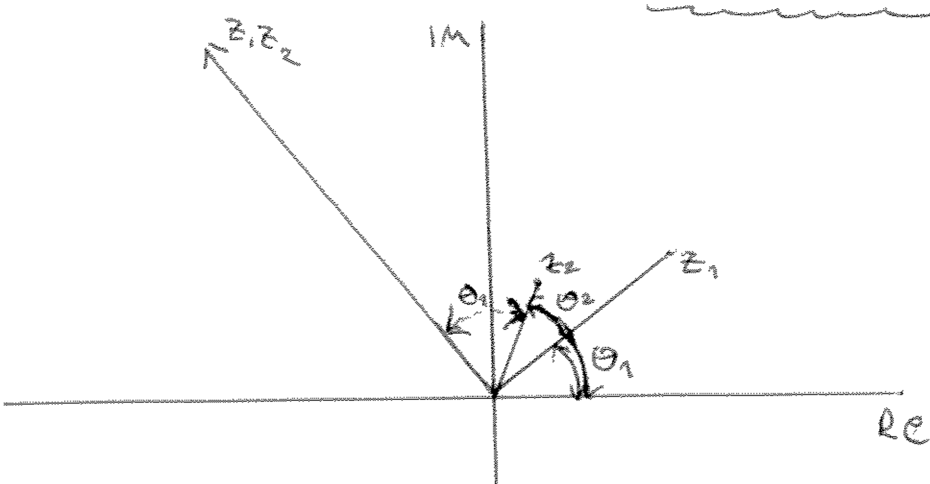
$\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$

$|z| = \sqrt{a^2+b^2}$

$\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

Kun $\alpha \in [0, 2\pi[$, $z \neq 0$

$z = |z|\cos\alpha + i|z|\sin\alpha = |z| e^{i\theta}$



Kompleksiluvulla kertominen voidaan ajatella kierroksi ja venytykseksi

3) $e^{rt} = e^{(a+ib)t} = e^{at+ibt} = e^{at}e^{ibt} = e^{at}(\cos bt + i\sin bt)$

$$\frac{de^{rt}}{dt} = ae^{at}e^{ibt} + e^{at}(-\sin bt \cdot b + i\cos bt \cdot b)$$

$$= ae^{rt} + e^{at}(i^2 b\sin bt + ib\cos bt)$$

$$= ae^{rt} + e^{at}(ib(\cos bt + i\sin bt)) = ae^{rt} + e^{at}\cdot ib\, e^{ibt}$$

$$= ae^{rt} + ibe^{rt} = (a+ib)e^{rt} = re^{rt}$$

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4) $A = \begin{pmatrix}0&1\\0&0\end{pmatrix} \Rightarrow A^2 = \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} = 0$

toisaalta $A^2 = VDV^{-1}VDV^{-1} = VDIDV^{-1} = VD^2V^{-1}$

Joten $VD^2V^{-1} = 0 \Rightarrow V^{-1}VD^2V^{-1} = 0$

siis $D^2V^{-1} = 0 \Rightarrow D^2V^{-1}V = 0 \Rightarrow D^2 = 0$

Koska oletus D muotoa $\begin{pmatrix}\alpha&0\\0&\beta\end{pmatrix} \Rightarrow D^2 = \begin{pmatrix}\alpha^2&0\\0&\beta^2\end{pmatrix} \Rightarrow \begin{cases}\alpha^2=0\\ \beta^2=0\end{cases}$

R.R koska silloin $A = VDV^{-1} = 0$ ↯

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5 $V = \begin{pmatrix}1&1\\1&-1\end{pmatrix} = V^T \Rightarrow VV^T = V^2 = V^TV$

$V^2 = \begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = \begin{pmatrix}2&0\\0&2\end{pmatrix}$ Josta voi päätellä, että

$V^{-1} = \frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = \begin{pmatrix}\frac12&\frac12\\\frac12&-\frac12\end{pmatrix}$

$VD = \begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}2&0\\0&1\end{pmatrix} = \begin{pmatrix}2&1\\2&-1\end{pmatrix}$

$VDV^{-1} = \begin{pmatrix}2&1\\2&-1\end{pmatrix}\begin{pmatrix}\frac12&\frac12\\\frac12&-\frac12\end{pmatrix} = \underline{\underline{\begin{pmatrix}\frac32&\frac12\\\frac12&\frac32\end{pmatrix}}}$