

Hint. Suppose that the functions $\varphi_k : [a, b] \rightarrow \mathbb{R}, k = 1, \dots, n$ are linearly independent functions in $C([a, b])$ and $f \in C([a, b])$ is given. The the least squares approximation of f in the subspace spanned by $\{\varphi_1, \dots, \varphi_n\}$ is given by

$$f^* = \sum_{j=1}^n c_j^* \varphi_j,$$

where c_j^* satisfy the normal equations, i.e.

$$\sum_{j=1}^n (\varphi_j, \varphi_k) c_j^* = (f, \varphi_k), k = 1, \dots, n.$$

If the functions $\{\varphi_1, \dots, \varphi_n\}$ form an orthogonal system, then these constants are denoted by c_k and given by

$$c_k = (f, \varphi_k) / (\varphi_k, \varphi_k), j = 1, \dots, n.$$