University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 03, 24.9.2012

`Problem sessions will be held on Monday at 16-18, B322.`

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. Suppose that the coefficients of a polynomial are known (for instance, generate 10 polynomials with random coefficients). Find the roots with the command "roots"and write the coefficients and the the real and imaginary parts of the roots in a file. Also plot the graph of the function.

2. (a) Prove that $\int_a^b \int_c^d xy\,dx\,dy = (d^2 - c^2)(b^2 - a^2)/4$ for $b > a, d > c$.

(b) Use the MATLAB function doubleint0.m on the www-page to compute this integral when $(a, b, c, d) = (0, 3, 0, 2)$ and tabulate the difference exact value minus numerical value when the number $m[n]$ of subdivisions in the $x[y]$ direction has the value $m = 10 : 20 : 90, n = 10 : 20 : 90$. You may do this as follows

```
exact=  (d^2- c^2)*(b^2 -a^2)/4;
for m=10:20:90
for n=10:20:90
    numer=doubleint0(a,b,c,d,m,n);
    fprintf('  %12.3e', numer-exact);
end
fprintf('\n')
end
```

3. The eigenvalues (=characteristic roots) of an $n \times n$ complex matrix $(a_{i,j})$ lie in the closed region of the z-plane consisting of all the disks

$$|a_{i,i} - z| \leq \sum_{j=1, j\neq i}^{n} |a_{i,j}|\,, j = 1, ..., n\,.$$

These are so called Gerschgorin disks. Check the validity of this statement as follows:

(a) For each `n=3:3:18` generate a random complex $n \times n$ matrix and compute its eigenvalues.

FILE: ~/mme11/teht/d03/d03.tex — 1. syyskuuta 2012 (klo 15.44).

(b) For each case plot the Gerschgorin disks and check visually that the statement holds.

4. The fixed point method for numerical solution of $f(x) = x$ when $f : \mathbb{R} \to \mathbb{R}$ is based on the fixed point iteration $x_{n+1} = f(x_n)$. This converges for all $x_0 \in [a, b]$ if there exists $c \in (0, 1)$ such that $|f'(x)| < c$. Choose a suitable $\lambda \neq 0$ such that the iteration $x_{n+1} = x_n + (1 - x_n - \sin x_n)/\lambda \equiv g(x_n)$ converges to the root of $1 - x - \sin x = 0$. Then use the method with a fixed λ to find the root.

5. A $m \times n$ matrix $A = (a_{ij})$ is called upper triangular if $a_{ij} = 0$ whenever $i > j$. Generate upper triangular 7×7 matrices and study experimentally whether (a) the product of two such matrices is again upper triangular, (b) an upper triangular matrix has an upper triangular matrix as the inverse (c) whether the determinant is always nonzero. Write a program to solve the upper triangular linear $n \times n$ system of equations.

6. MATLAB has a built-in function magic which with call magic(n) generates an $n \times n$ matrix with positive integer entries such that the row and column sums are all constant (such a matrix is called a magic matrix). Is it true that the inverse of a magic matrix also is a magic matrix (note the entries of the inverse are no longer integers so it would be helpful to use "format rational"to see the exact entries, not only their numerical approximations). If a is a magic matrix, is it true that also a^2 is?