University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 02, 17.9.2012

Problem sessions will be held on Monday at 16-18.

1. A data set $(x_j, y_j), j = 1, ..., m$, and a fixed point (s, t) is given. Consider the lines $y = t + k(x - s)$ through this point and for varying parameter k form the sum

$$A(k) = \sum_{j=1}^{m} (y_j - t - k(x_j - s))^2 .$$

Write the formula for the derivative $A'(k)$, and solve the linear equation $A'(k) = 0$ for k. With this particular value of k we get the least square (LSQ) line fitted to our data set through the point (s, t). Generate some synthetic data [e.g. $x_j = 0.5 * j, y_j = 0.3 * x_j + 0.1 * \sin(30 * x_j), j = 1, ..., 20,$] plot the data and the LSQ-line through the point $(0, 0)$ and check visually whether the line fits nicely to the data set.

**2.** P. J. Myrberg (1892–1976) has published the following algorithm for the computation of the square root in his paper in Ann. Acad. Sci. Fenn. Ser. A I 253 (1958), 1-19. Fix $z \in \mathbb{C} \setminus \{1\}$ and let $q_0 = (z+1)/(z-1), q_{k+1} = 2q_k^2 - 1, k = 0, 1, ...$ Then $\sqrt{z} = \Pi_{k=0}^{\infty}(1 + 1/q_k)$. Carry out MATLAB tests to check this claim in the following cases
   (a) z is real and > 1,
   (b) z is real and in $[0, 1]$.
   (c) z is complex [Hint: Generate a random complex number w and set $z = w * w$ and see whether the algorithm gives you w.]

3. The use of the Trapezoid formula for numerical integration of a tabulated function $f(x_i) = y_i$ is based on the formula

$$s = \sum_{i=1}^{n} (x_{i+1} - x_i)(y_{i+1} + y_i)/2,$$

where the tabulated values are $(x_i, y_i), i = 1, \ldots, n + 1$, and $x_i < x_{i+1}$.

Trapezoidal rule for numerical integration
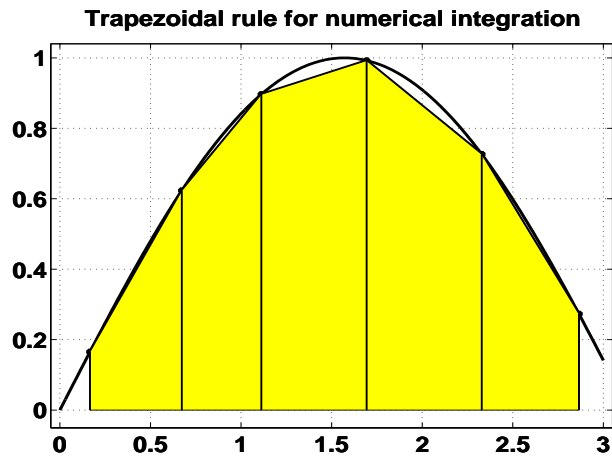
(a) Show that if $f(x) = \sum_{j=1}^{m} c_j \sin(d_j * x)$, then by high school calculus

$$\int_a^b f(x)dx = \sum_{j=1}^{m}(c_j/d_j)(\cos(d_j * a) - \cos(d_j * b)).$$

(b) Generate coefficient vectors c and d [e.g. c=3*rand(1,5); d= 1+10*rand(1,5)] and use the exact formula in part(a) to compute the integral $\int_0^1 f(x)dx$ and also compute the value by the trapezoid formula and print the error when the points $x_j = (1/n) * j, j = 0, 1, ..., n$, are used in the trapezoid formula with $n = 20 : 10 : 200$.

4. The compositions of W. A. Mozart (1756–1791) are identified with Köchel numbers which indicate the order of composition. Some of the numbers and the years are

Number	Year
1	1761
75	1771
155	1772
219	1775
271	1777
351	1780
425	1782
503	1786
575	1789
626	1791

Use this data to estimate the year of composition of Sinfonia Concertante in E-flat major, Köchel number 364.

5. Fix $a > 0$. It is well-known that if $0 < x_0 < 2/a$ the recursive sequence defined by $x_{k+1} = x_k(2 - ax_k)$ converges to $1/a$.

(a) Write a MATLAB function to compute the reciprocal with this method.

(b) Apply the method to several test cases (say to $0.1*k, k = 1 : 1000$,) and report the error.

6. Numerical analysis textbooks often point out that the quadratic formula $x_{1,2} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$ may lead to distorted results when $|4ac|$ is very small. Plan a MATLAB experiment to justify this remark and report the errors in the test cases you have used. (Hint. Choose $a = b^2/(4c) \cdot 10^{-m}$, $m = 3, \ldots, 15$.)