Introduction to Probabilistics, Exercises I

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1. Five Americans, seven British and three Germans are seated in a random order around a table. What is the probability that all the Americans sit together? 1% What is the probability that all the Americans and all the British sit together? 2% What is the probability that all the Americans, all the British and all the Germans sit together? 2%

2. In this problem, which is often called the birthday problem, it is required to determine the probability p that at least two people in a group of k people will have the same birthday, that is, will have been born on the same day of the same month but not necessarily in the same year. Assume that the birthdays of the k people are unrelated (in particular, assume that twins are not present) and that each of the 365 days of the year is equally likely to be the birthday of any person in the group. Furthermore, ignore the fact that the birth rate actually varies during the year and assume that anyone actually born on February 29 will consider his birthday to be another day, such as March 1. 5%

3. We now consider the Monty Hall dilemma. Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number, and the host, who knows what's behind the doors, opens another door which has a goat. He says to you, "Do you want to pick the other remaining door?" Is it to your advantage to switch your choice of doors? 4% What if there was n different doors and only one of them was a car? 1% -Note that in this exercise car is implicitly privileged over a goat which by no means is inclusive(!)

4. Your favorite team participates in a knock-out system that consists of four rounds. If your team has reached round i, it will survive this round with a given probability of p_i for $i = 1, \ldots, 4$. After the competition, you are informed that your team is not the final winner. This is the only information you get about the tournament. What is the probability that your team was eliminated in round i? 5%

5. A football team consists of 20 offensive and 20 defensive players. The players are to be paired in groups of 2 for the purpose of determining roommates. If the pairing is done at random, what is the probability that there are no offensive-defensive roommate pairs? 2% What is the probability that there are $2i$ offensive-defensive roommate pairs, $i = 1, 2, \ldots, 10$? 3%