Integral equations

HW 1

1. Solve the Volterra equation

$$\phi(s) - \int_0^s (s-t)\,\phi(t)\,dt = 2s$$

2. Solve the Volterra equation

$$\phi(s) - 4\int_0^s (s-t)\,\phi(t) = s^3$$

3. Reduce the initial value problem

$$y^{(3)} + 2xy = 0,\ y(0) = y'(0) = 0, y''(0) = 1$$

to a Volterra equation of second kind.

4. Solve the Volterra equation of the first kind

$$\int_1^s (s+t)\phi(t)\,dt = s^3 - 1.$$

5. Consider a Volterra equation of the first kind

$$\int_a^s K(s,t)\phi(t)\,dt = f(s) \tag{0.1}$$

where K and f are continuous. Assume $K(s,s) = 0$ for all $s \in [a,b]$ and that K has continuous partial derivatives with respect to s up to second order. Formulate and prove a solvability result for (0.1).

6. Let K be a continuous kernel. Consider the iterated kernels

$$K^{(1)}(s,t) = K(s,t), \quad K^{(n)}(s,t) = \int_t^s K(s,r)K^{(n-1)}(r,t)\,dr$$

defined in the lectures. Prove that we also have

$$K^{(n)}(s,t) = \int_t^s K^{(n-1)}(s,r)K(r,t)\,dr.$$

Hint: Use induction on n.