Department of mathematics and statistics
Differential Equation II
Compensating Course Exam 17.1.2013

Remark. A candidate is allowed to use a short abstract of size A4.

1. (a) (4 points) Reduce the following differential equation of second degree to a system of first degree:

$$\ddot{x}(t) + 3t\,\dot{x}(t) + x(t)^2 = \sin t.$$

(b) (2 points) Write for that system an initial condition related to the initial condition $x(1) = -1,\ \dot{x}(1) = 0$ of the original equation.

2. Find in $\mathbf{R}$ a fundamental solution set to the following homogeneous system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & -5 \\ 1 & 3 \end{bmatrix} \mathbf{x}(t).$$

3. Give a general solution to the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & 0 \\ -1 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

4. Conserning the following autonomous system determine its critical points, their qualities (stable or unstable) and additionally calculate trajectories of its solutions in the xy phase plane:

$$\begin{aligned} \dot{x}(t) &= x(t)^2 - 1 \\ \dot{y}(t) &= x(t)y(t). \end{aligned}$$

Remark. It is neither necessary nor worth drawing trajectories.