Department of mathematics and statistics
Differential Equation II
Course Exam 11.12.2012

Remark. A candidate is allowed to use a short abstract of size A4.

1. Find a fundamental solution set in $\mathbf{R}$ to the following homogeneous system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \mathbf{x}(t).$$

2. (a) (4 points) Calculate three first Picard's approximates y_0, y_1 and y_2 to the initial value problem

$$y'(x) = \cos x \sin y, \quad y(-1) = \pi.$$

(b) (2 points) Give also the approximate y_{1000}.

3. Give a general solution to the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1/2 \\ -2 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -3e^{-t} \\ 2e^{-t} \end{bmatrix}.$$

4. Reduce the (scalar) differential equation

$$\ddot{x}(t) = -2\dot{x}(t) + x(t)^2 + x(t) - 2$$

of second order to a system of first order and determine critical points of that autonomous pair and give also their qualities (stable or unstable).