Differential Equations II

Exercise 5, fall 2012

1. Find by the matrix method (applying generalized eigenvectors) a fundamental solution set in $\mathbf{R}$ for the system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t), \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$

2. Consider again the previous system. Outline briefly, how else one can find a fundamental solution set for it. Don't need to carry out the ideas.

3. Determine critical points and their qualities (stable or unstable) of the following autonomous system:

$$\begin{aligned} \dot{x} &= 2y - 2 \\ \dot{y} &= -x + 2y. \end{aligned}$$

4. Determine critical points and their qualities of the following autonomous system:

$$\begin{aligned} \dot{x} &= -2x + y + 6 \\ \dot{y} &= x - 2y + 3. \end{aligned}$$

5. Determine critical points and their qualities of the following autonomous system:

$$\begin{aligned} \dot{x} &= x^2 - y \\ \dot{y} &= 2 - x^2 - y^2. \end{aligned}$$

6. Determine critical points of the following autonomous system:

$$\begin{aligned} \dot{x} &= (x+1)(y-2) \\ \dot{y} &= x^2 - x - 2. \end{aligned}$$

What does the Poincaré's theorem tell about their qualities? Determine also trajectories of the system.