Differential Equations II
Exercise 4, fall 2011

1. Find a fundamental solution set for the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, where

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -8 & 14 & -7 \end{bmatrix} \in \mathbf{R}^{3\times 3}.$$

Determine also quality of the equilibrium solution $\mathbf{0}$ (stable or unstable).

2. Find a fundamental matrix for the homogeneous system $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \in \mathbf{R}^{2\times 2}.$$

3. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} -2 \\ 3 \end{bmatrix},$$

by using variation. What a direct try should easier lead to a goal?

4. Solve the linear system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t), \quad A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad \mathbf{f}(t) = \begin{bmatrix} -\cos t \\ -\sin t \end{bmatrix},$$

by using an appropriate direct try. It leads to an ordinary 4×4 linear, algebraic system of parameters.

5. Find for the following system a fundamental solution set in $\mathbf{R}$ by the matrix method, which applies generalized eigenvectors:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 1 \\ -1 & -3 \end{bmatrix} \mathbf{x}(t).$$

A tip. Equations (5.31) and (5.32) in the lecture material.