Differential Equations II
Exercise 3, fall 2012

1. Reduce the differential equation $y'' - \frac{x}{3}\,y^{(3)} + xyy' - 2x^3 = 0$ to a system of first order and of normal form. Is that system linear?

2. Show that the pair $\big(\mathbf{x}_1(t), \mathbf{x}_2(t)\big) = \Big([1\ e^t]^T,\ [e^{-t}\ 2]^T\Big)$ of functions is a fundamental solution set in $\mathbf{R}$ to the linear homogeneous 2×2-system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & -e^{-t} \\ 2e^t & -1 \end{bmatrix} \mathbf{x}(t).$$

3. Look for the following homogeneous 2×2-system a fundamental solution set in $\mathbf{R}$ and give also a general solution:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 2t & 3t^2 \\ 0 & 2t \end{bmatrix} \mathbf{x}(t).$$

A tip. One of the equations luckily happens to be simple.

Below (and in what later follows) the notation $\mathbf{0} = (0, \cdots, 0) \in \mathbf{R}^n$ will be used.

4. Give a general solution in $\mathbf{R}$ to the following homogeneous 2×2-system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix} \mathbf{x}(t).$$

Briefly, is the trivial solution $\mathbf{x}(t) \equiv \mathbf{0}$ a stable or unstable equilibrium solution?

5. Look for the following homogeneous system a fundamental solution set in $\mathbf{R}$:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \mathbf{x}(t).$$

Briefly, is the trivial solution $\mathbf{x}(t) \equiv \mathbf{0}$ a stable or unstable equilibrium solution?

6. Look for the following homogeneous system a fundamental solution set in $\mathbf{R}$:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \mathbf{x}(t).$$

Briefly, is the trivial solution $\mathbf{x}(t) \equiv \mathbf{0}$ a stable or unstable equilibrium solution?