Differential Equations II
Exercise 2, fall 2012

1. Solve by elimination method the following linear system of first order:

$$\mathbf{y}'(x) = \begin{bmatrix} -2 & 1/2 \\ 2 & -2 \end{bmatrix} \mathbf{y}(x), \quad \mathbf{y} = (y_1, y_2).$$

A tip. Write first the pair in a traditional form by means of functions y_1 and y_2.

2. Reduce the following scalar equations to systems of first order:
(a) $y'' + \sin x\, y' + y = \cos x$,
(b) $y^{(4)} + x^4 y = \sin x$.

3. (a) Reduce the following system to a system of first order and normal form:

$$\begin{aligned} \dot{y} &= f(t, x, y) \\ \ddot{x} &= g(t, x, y, \dot{x}). \end{aligned}$$

(b) What about the second equation being

$$\ddot{x} = g(t, x, y, \dot{x}, \dot{y})?$$

4. Let y be a (maximal) solution to the initial value problem

$$y' = e^x \sin x \cos y, \quad y(0) = 0.$$

Show by the global EU Theorem 4.6, that y is defined on the whole $\mathbf{R}$.
A tip. The Mean Value Theorem.

5. The same as above, but now by the "Go Away" Theorem 4.7.
A tip. Trivial solutions.

6. A general solution to Problem 2 in Exercise 1 has four parameters. Show exactly: that solution gives all the solutions to the involved differential equation.
A tip. Theorem 5.4, but if you prefer to reduce to a system, Theorem 5.3 (or 5.5).