Differential Equations II
Exercise 1, fall 2012

1. Solve the following linear differential equation of second order:

$$y'' - 3y' - 4y = 4x + 7.$$

2. Solve by adapting the theory of linear equations of second order the following homogeneous equation of fourth order:

$$y^{(4)} - 7y'' + 6y = 0.$$

Does not need to give exact reasons you have all the solutions for.

3. Calculate four first Picard's approximations for the initial value problem

$$y' = y + 1, \quad y(0) = -1,$$

and compare to the exact solution.

4. Calculate four first Picard's approximations for the initial value problem

$$y' = y + 1, \quad y(0) = 0,$$

and compare to the exact solution.

5. Is the function $f(x, y) = x^2 \sin y$ in the set $I \times J$ uniformly Lipchitz continuous with respect to y, when
(a) $I = \mathbf{R}$ and $J = [0, 1]$,
(b) $I = [0, 1]$ and $J = [0, 1]$,
(c) $I = [0, 1]$ and $J = \mathbf{R}$?
Arguments. If it does, give also (any) valid Lipschitz constant.

6. Where in domains of $\mathbf{R}^2$ (as large as possible) the differential equation

$$y' = f(x, y) = \sqrt[5]{(y-1)^2}$$

satisfies the conditions of the local EU-Theorem 4.4? Short arguments.
Remark. The equation is defined in the whole plane $\mathbf{R}^2$, indeed.