Differential Equations I

Exercise 4, fall 2012

1. Find the curves of the form $y = y(x)$ such that for all x_0 a tangent line, set at $\big(x_0, y(x_0)\big)$, intersect the x-axis at $\big(x_0 + x_0^2/k,\, 0\big)$, where $k \neq 0$ is a constant.

2. Let temperatures of a body and its environment be functions of time t, $T_1 = T_1(t)$ and $T_2 = T_2(t)$, and suppose they interact so that at each moment their ranges of change are proportional to the difference $T_1(t) - T_2(t)$ between temperatures, $a < 0$ and $b > 0$ as proportional constants (so called Newton's law of cooling).

(a) Form a pair of differential equations for the functions T_1 and T_2.

(b) Can you solve that?

A tip. (b) Eliminate the pair to one equation.

3. Solve the equation

$$y' = (x + y + 2)^2.$$

4. Solve the equation

$$x^2 - xyy' + y^2 = 0.$$

5. Solve the equation

$$\frac{dy}{dx} = \frac{-3x + y + 4}{x + 3y + 2}.$$

6. A falcon flies at a high of 200 meters, when it notices a rabbit precisely below. At the same time the rabbit recognizes the falcon and speeding up like a lightning it runs at the rate of 10 m/s straight at his home hole. Constantly the falcon falls straight at the rabbit at the rate of 35 m/s. Does the falcon catch up the rabbit before it arrives his hole, when that is at a distance of 60 meters?