**Differential Equations I**

Exercise 1, fall 2012

1. Name which of following ones are ordinary differential equations, name also an unknown function and give order of the equation:

$$(a)\quad \ddot{x} = (t + x - 2)^3, \quad (b)\quad xy + \sin(xy')^2 - 5y' = 1,$$
$$(c)\quad y = \frac{\partial z}{\partial x} - 2xy + yz, \quad (d)\quad y'(x) = y(x^2).$$

In the last equation all in brackets belongs to an argument of the function.

2. Of following ones name and solve separable differential equations:

$$(a)\quad y' = 2x - 2xy, \quad (b)\quad y' = \cos(x - y) \quad (c)\quad y' = 2x - 2y.$$

Nonseparability is not needed to prove.

3. Solve the equation

$$1 + 2x - 2yy' = 0$$

with the initial conditions

$$(a)\quad y(0) = -2, \quad (b)\quad y(0) = 0.$$

4. Solve the initial value problem (IVP)

$$y' = (y - 2)(y - 1), \quad y(0) = 0.$$

What is a maximal solution interval of it?

5. Does the existence and uniqueness Theorem (EU-teorem) 1.2 guarantee a unique solution to the IVP

$$(a)\quad y' + \cos^3 y = \sin^4 x, \quad y(\pi) = 0, \quad (b)\quad y' + \cos^3 y = \sqrt{\sin x}, \quad y(\pi) = 0,$$
$$(c)\quad \text{Problem 3a}, \quad (d)\quad \text{Problem 3b?}$$

6. Of which type is the equation at point 2c, that is $y' = 2x - 2y$?

A tip. Partial integration in the end.