Linear algebra and matrices II
Department of mathematics and statistics
Autumn 2011
Exercise sheet 5, MATLAB-exercises 18-19

Using formulas to find out the determinant of large matrices is computationally hard. This kind of calculations requires usually over $n!$ floating point calculations, where n is the amount of rows of the matrix. For example calculating a 29×29 matrix's determinant using the Ukko cluster[1] of the CS department would take three times the age of the universe[2]. Lukily MATLAB doesn't use formulas and it takes it less than a second to calculate a determinant, using an algorithm similar to Gauss-Jordan's elimination, which computationally easy. In the next exercises we'll have a look at how MATLAB'S `det` function works.

The questions you're supposed to answer to are in **boldface**.

18.

1. Open MATLAB (check Linear algebra I, exercise sheet 4, exercise 30).
2. Generate a random 29×29 matrix and find out its determinant using the command:
   `det(randn(29))`
   The time required for the calculation should be less than the age of the universe.

Ruvetaan selvittämään algoritmia jolla MATLAB urakasta suoriutuu:

3. Substitute into the variable `matrix` the 6×6 matrix below with the command:

```
matrix =
[3 2 4 5 1 1;
1 4 3 5 2 3;
3 5 1 3 4 1;
1 4 3 4 2 1;
3 5 1 2 4 1;
1 4 2 1 5 1]
```

[1] Thanks to Pekka Mikko for the information about the computational power of the Ukko cluster.

[2] 13,7 billion years.

4. Find the LU decomposition for the matrix and substitute it into the variables `toptriangle`, `bottomtriangle` ja `permutation` with the command:
   `[bottomtriangle,toptriangle,permutation] = lu(matrix)`

   (LU decomposition: `http://en.wikipedia.org/wiki/LU_decomposition`)

5. Calculate: `permutation*matrix - bottomtriangle*toptriangle`

6. **Basing on theorem 5.22. what do we know about the matrix `matrix` 's determinant?**
   Hint: use the previous calculation and build up and equation in which you can solve for the value of the determinant of matrix `matrix`.

19.

7. **What do we know now basing on theorem 5.3.9? For instance what is matrix `bottomtriangle` 's determinant?**

8. If the square matrix A has been obtained by switching two rows of a matrix B then $\det(A) = -\det(B)$. What do we know now about the matrix `permutation` 's determinant?
   Hint: what is the determinant of a unit matrix, according to example 5.3.9.?

9. The vector corresponding to the diagonal of a square matrix `A` can be obtained in MATLAB typing the command: `diag(A)`. Substitute such vector for `toptriangle` into the variable `diagToptriangle`.

10. In MATLAB you can obtain the product of the elements of vector `B` by the command `prod(B)`. Substitute into the variable `prodDiagToptriangle` the product of the elements of `diagToptriangle`.

11. **Calculate: `det(matriisi)`. Why is it different from `prodDiagToptriangle`?**
    Hint: part 8.

The answers to these last questions should clear up how the function `det` works in MATLAB. Further readings:
`http://www.mathworks.se/help/techdoc/ref/det.html`