

Structure of tomographic X-ray data

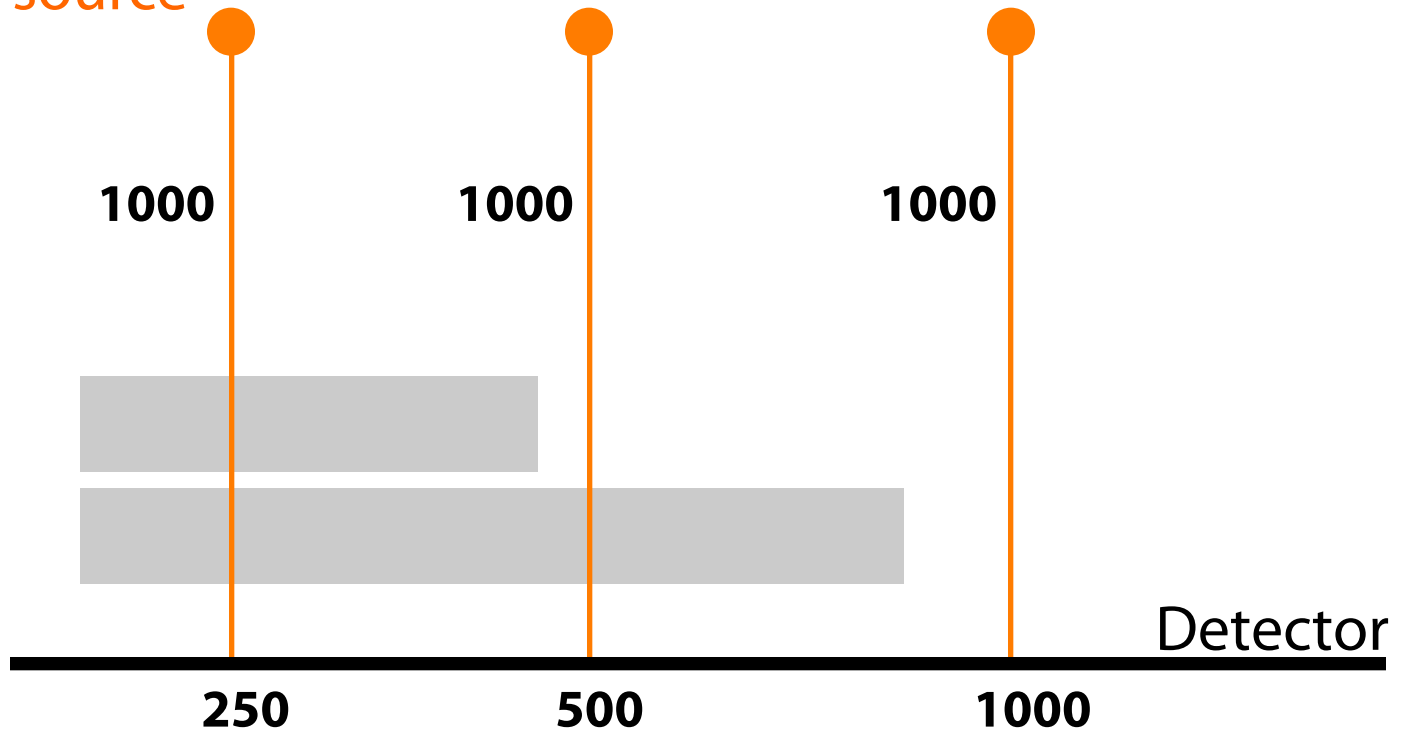
A scenic view of a rocky coastline. The foreground shows a sandy beach with gentle waves washing onto it. In the middle ground, a large, white-capped wave is breaking, creating a spray of water. The background features rugged, golden-brown rock formations and cliffs extending into the distance under a clear blue sky. The overall scene is bright and clear, suggesting a sunny day.

Samuli Siltanen



X-ray images as measurements

X-ray source



250

500

1000

Logarithm

5,5

6,2

6,9

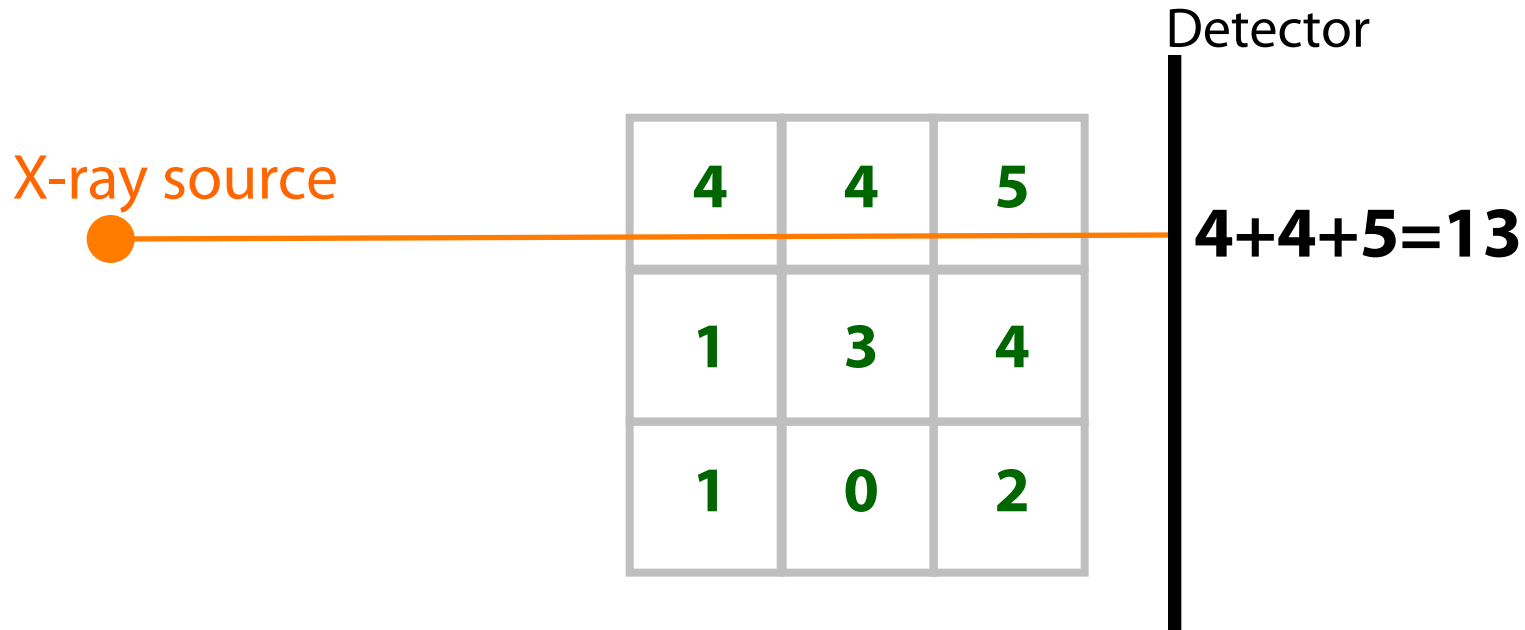
Density

1,4

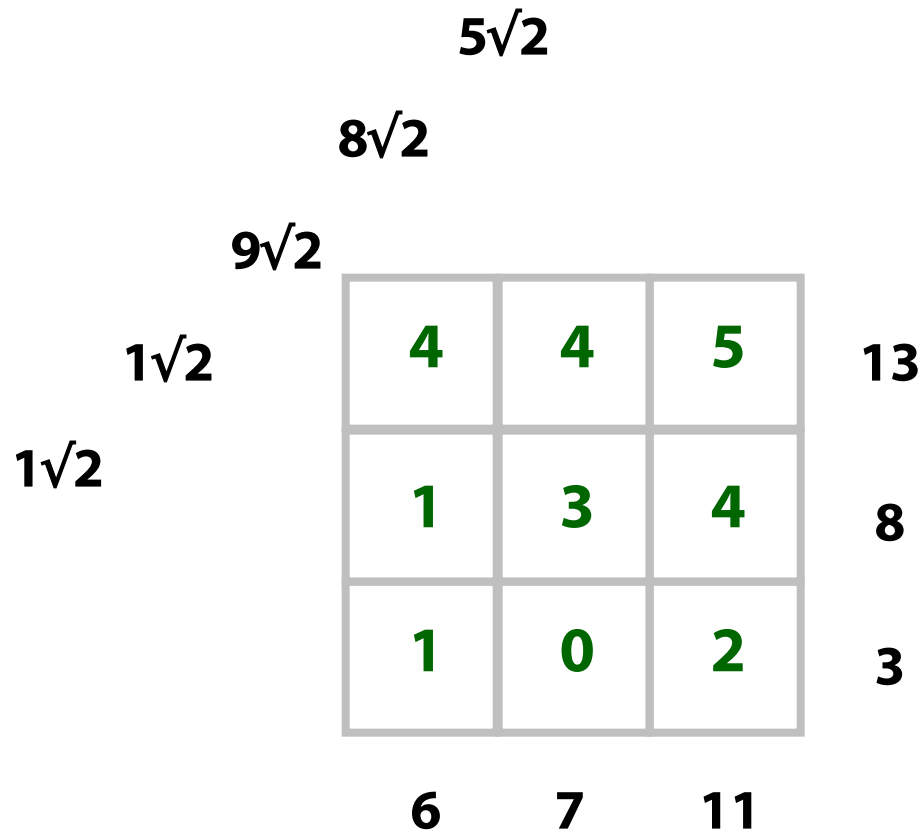
0,7

0,0

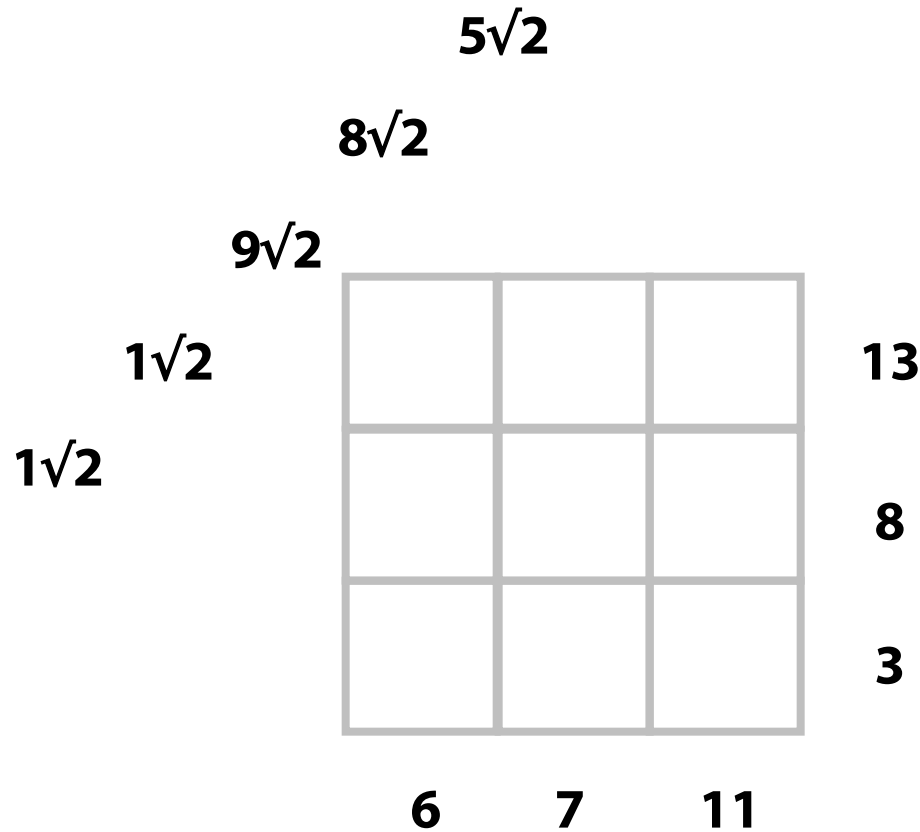
Every X-ray measures the sum of attenuation through tissue



Direct problem of tomography is to find the radiographs from given tissue

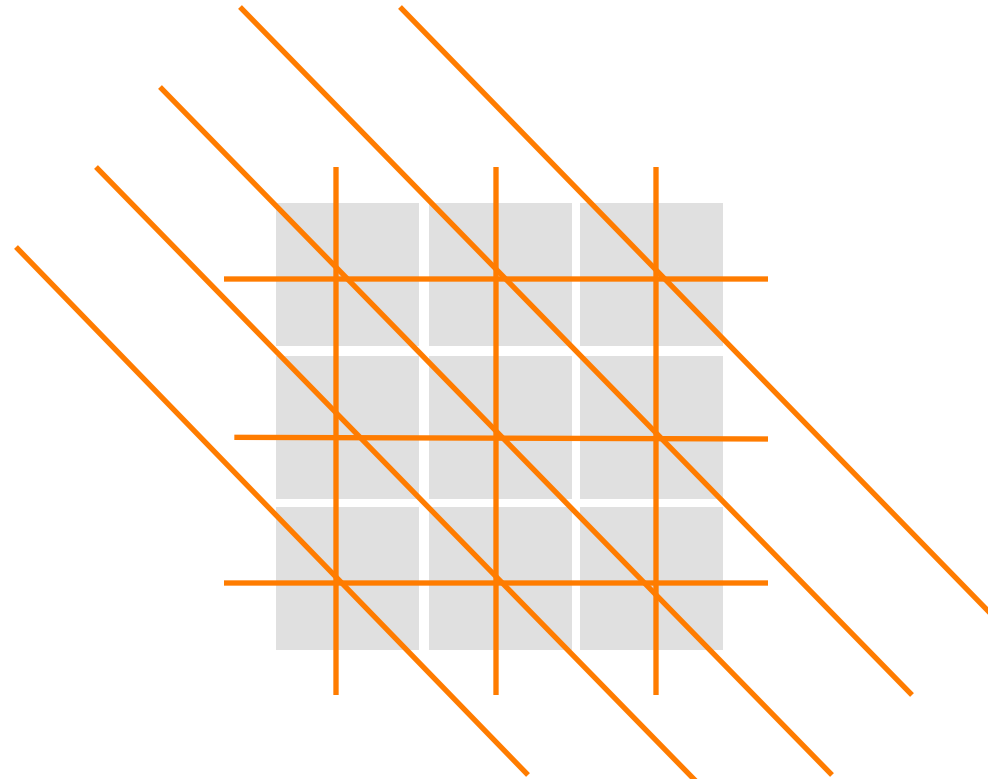


Inverse problem of tomography is to find the tissue from radiographs

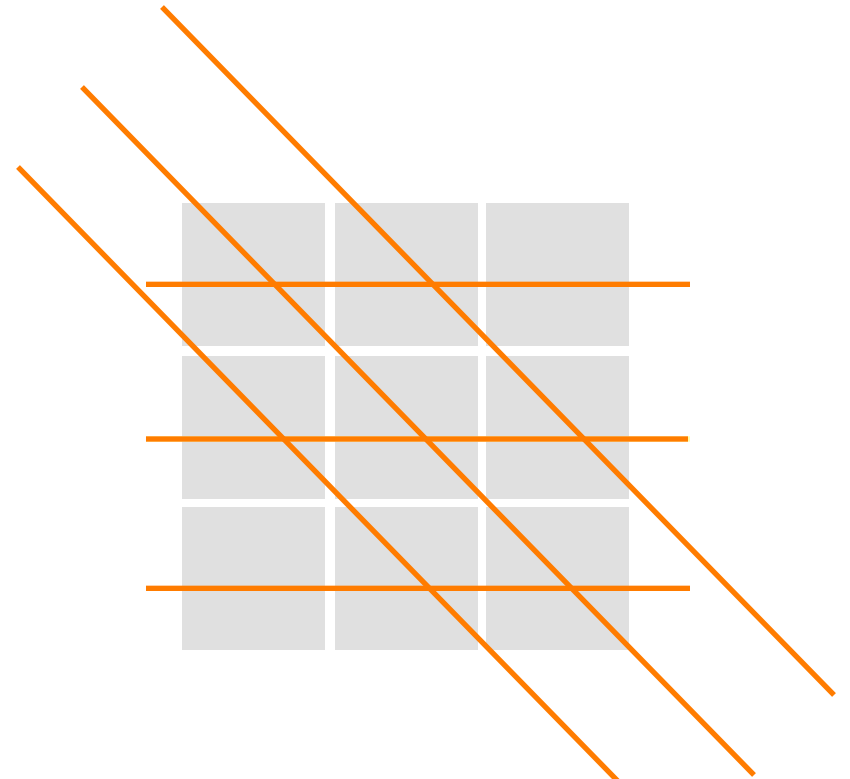


9 unknowns, 11 linear equations

The limited angle problem is harder than the full angle problem



9 unknowns,
11 linear equations



9 unknowns,
6 linear equations

In limited angle 3D imaging there are many tissues matching the radiographs

	$8\sqrt{2}$											
	$9\sqrt{2}$											
$1\sqrt{2}$	<table border="1"><tr><td>4</td><td>4</td><td>5</td></tr><tr><td>1</td><td>3</td><td>4</td></tr><tr><td>1</td><td>0</td><td>2</td></tr></table>	4	4	5	1	3	4	1	0	2		13
4	4	5										
1	3	4										
1	0	2										
			8									
			3									

5	6	2
1	5	2
4	0	-1

9	1	3
1	0	7
3	0	0

a priori information is needed!

We write the reconstruction problem in matrix form and assume Gaussian noise

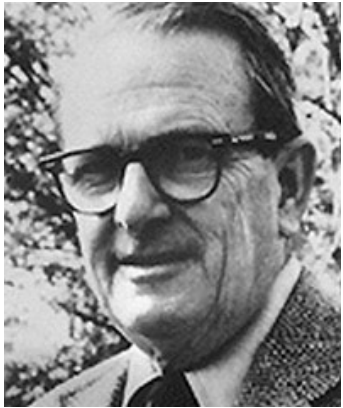
			m3		
	m2				
m1		x1	x4	x7	m4
		x2	x5	x8	m5
		x3	x6	x9	m6

$$x = [x_1, x_2, \dots, x_9]^T$$
$$m = [m_1, m_2, \dots, m_6]^T$$

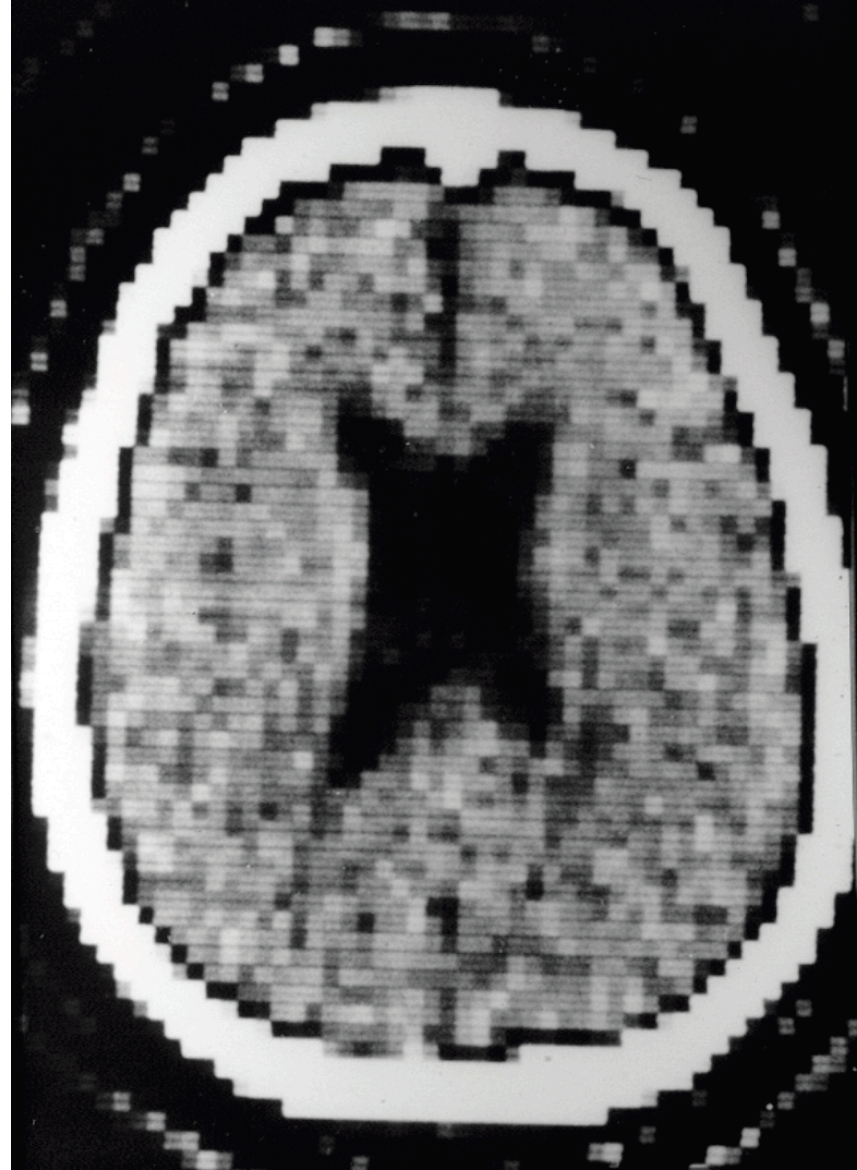
$$Ax = m$$

We consider the measurement model $m = Ax + \varepsilon$ with additive Gaussian noise ε of standard deviation σ .

Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography



Nobel prize was awarded to Hounsfield (top) and Cormack in 1979.



Using a reconstruction algorithm, inner structure in the slice is revealed

This is called computerized tomography (CT).

