EVOLUTION AND THE THEORY OF GAMES

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Exercises 18-11-2011

7. Show that, in a game with two pure strategies, the ESS conditions are equivalent to

$$\pi_1(x', x) < \pi_1(x, x)$$

or

$$\pi_1(x', x) = \pi_1(x, x) \text{ and } \pi_1(x', x') < \pi_1(x, x')$$

for every *pure* strategy $x' \neq x$.

8. Calculate all evolutionarily stable strategies (pure and mixed) for the Hawk-Dove game

	H	D
H	$\frac{1}{2}R - \frac{1}{2}C$, $\frac{1}{2}R - \frac{1}{2}C$	R, 0
D	0, R	$\frac{1}{2}R$, $\frac{1}{2}R$

for **(a)** $V > C$, **(b)** $V = C$ and **(c)** $V < C$.

9. Extend the Hawk-Dove game with a third strategy called "Retaliator" (R) who plays Dove against Dove but Hawk against Hawk, and also Hawk against itself. Give the payoff matrix of the Hawk-Dove-Retaliator game and calculate all ESSs for **(a)** $V > C$ and **(c)** $V < C$.

10. Extend the Hawk-Dove game with a third strategy called "Bully" (B) who plays Hawk against Dove but Dove against Hawk, and also Dove against itself. Give the payoff matrix of the Hawk-Dove-Bully game and calculate all ESSs for **(a)** $V > C$ and **(c)** $V < C$.

11. Formulate and analyze the Retaliator-Bully game.

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12. Consider the Rock-Paper-Scissors game with the payoff matrix

	R	P	S
R	$-\varepsilon$, $-\varepsilon$	-1 , 1	1 , -1
P	1 , -1	$-\varepsilon$, $-\varepsilon$	-1 , 1
S	-1 , 1	1 , -1	$-\varepsilon$, $-\varepsilon$

for $\varepsilon > 0$. Show that $x = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is an ESS. Are there any other ESSs?