Luecking's theorem and finite rank Toeplitz operators

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Consider the Bergman space A^2 consisting of analytic functions of the unit disk having finite L^2 norm. For a Borel measure, we define the Toeplitz operator on A^2 by

$$T_\mu f(z) = \int \frac{f(w)d\mu(w)}{(1 - z\overline{w})^2}.$$

One of the most notable recent results in the theory is the complete characterization of finite rank Toeplitz operators by D.Luecking. According to this result, a Borel measure μ generates a finite rank Toeplitz operator if and only if it is a linear combination of point masses. This, in particular, says that no L^1 function generates finite rank Toeplitz operators.

The proof is quite elegant and (surprisingly enough) requires almost no knowledge of Toeplitz operators; we actually prove a more general result on the space of analytic polynomials.