Model Theory
Exercise 5
Hints.

1. Model completeness: Suppose ~~[illegible]~~ $\mathfrak{B} \subseteq \mathfrak{C}$ are models of $Th(\mathfrak{A})$.
By 4.5 it is easy to see that w.o.l.g. we may assume that $\mathfrak{C}$ is countable.
Also $Th(\mathfrak{A})$ is ω-categorical and so we may assume that $\mathfrak{B} = \mathfrak{A}$.
But then it is easy to see that there is an isomorphism $f : \mathfrak{C} \to \mathfrak{A}$ s.t. for all $x < n$, $f(x) = x$, where $n \in \mathbb{N}$ can be chosen as large as we want.
Clearly this implies $\mathfrak{B} \preceq \mathfrak{A}$.

One can not eliminate quantifiers from

$\exists y \, (P_0(y) \wedge R(x,y))$ (think what are

atomic formulas in one variable).

2. $L = \{c_0, c_1\}$, $T = \{\varphi_n \mid n \in \mathbb{N}\}$

$$\varphi_n = \exists v_0 \dots \exists v_n \Big(\bigwedge_{i<j\leq n} \neg v_i = v_j \Big),$$

3. Check the definitions

4. $P_1(x) = 0 \vee \dots \vee P_n(x) = 0$ is

the same as $\prod_{i=1}^{n} P_n(x) = 0$. Then

use propositional logic.

5. For AP: Suppose $\mathfrak{A} \subseteq \mathfrak{B}, \mathfrak{C}$.

Choose $\mathfrak{D} \supseteq \mathfrak{C}$ s.t. $\dim(\mathfrak{D}) > \dim(\mathfrak{B})$

Let $I_0 \subseteq \mathfrak{A}$ be a basis of $\mathfrak{A}$,

$I_0 \subseteq I_1 \subseteq \mathfrak{B}$ a basis of $\mathfrak{B}$ and

$I_0 \subseteq I_2 \subseteq \mathfrak{D}$ a basis of $\mathfrak{D}$. Chosen

an injection $f: I_1 \to I_2$ s.t. $f \upharpoonright I_0 = id$.

Then f extends to an embedding

$g: \mathfrak{B} \to \mathfrak{D}$, such that $g \upharpoonright \mathfrak{A} = id$.

For the rest as in the case of T_{acf_0}.