Model Theory
Exercise 2

1. If $\mathfrak{D} \models T$ has power $\kappa > \omega$, then $\mathfrak{D}$ consists of one copy of ~~?~~ and κ copies of $(\mathbb{Z}, S)$, $S(x) = x+1$.

2. E.g. Let g and h be as in 3.3 for $t(x)$ and $u(x)$ and $g_1, \ldots, g_n$.

$$\mathfrak{A} = \prod_{\eta \in X} \mathfrak{A}_\eta / U \models t = u(g_1/U, \ldots, g_n/U)$$

iff $\{\eta \in X \mid g(\eta) = h(\eta)\} \in U$

iff $\{\eta \in X \mid t^{\mathfrak{A}_\eta}(g_1(\eta), \ldots, g_n(\eta)) = u^{\mathfrak{A}_\eta}(g_1(\eta), \ldots, g_n(\eta))\} \in U$

iff $\{\eta \in X \mid \mathfrak{A}_\eta \models t = u(g_1(\eta), \ldots, g_n(\eta))\} \in U$.

3. By Fact 4.4. it is enough to show the claim for theories Σ and sentences φ.

Then $\Sigma \models \varphi$ iff $\Sigma \cup \{\neg\varphi\}$ is inconsistent

Thus there is finite $\Sigma' \subseteq \Sigma$ s.t.

$\Sigma' \cup \{\neg \varphi\}$ is inconsistent by 3.6.

But then $\Sigma' \models \varphi$.

4. ~~Suppose~~ For a contradiction, suppose that for n, k and p there is no such m.

To simplify the notation we suppose $k = 2$.

Choose $L = \{c_i \mid i \in \omega\} \cup \{<, R\}$

($<$ is an ordering and R is intended to be $f^{-1}(0)$)

Look at the theory T which says

"$<$ is a linear ordering of the univers"

for each $i \in \omega$, "c_i is the $i+1^{th}$ element in the linear ordering $<$"

~~R is symmetrill~~

$$g = \neg \exists x_1 \dots \exists x_p \Big(\bigwedge_{1 \le i < j \le p} x_i < x_j \wedge \Big[$$

$$\forall y_1 \dots \forall y_n \Big(\Big(\Big(\bigwedge_{1 \le i \le n} \bigvee_{1 \le j \le p} y_i = x_j \Big) \wedge \Big(\bigwedge_{1 \le i < j \le n} y_i < y_j \Big) \Big)$$

$$\rightarrow R(y_1, \dots, y_n) \Big) \vee \forall y_1 \dots \forall y_n \Big(\Big(\Big($$

$$\bigwedge_{1 \le i \le n} \bigvee_{1 \le j \le p} y_i = x_j \Big) \wedge \Big(\bigwedge_{1 \le i < j \le n} y_i < y_j \Big) \Big) \rightarrow$$

$$\neg R(y_1, \dots, y_n) \Big) \Big) \Big]$$

Then T is consistent by compactness and if $\mathfrak{A} \models T$ then we can find a counter example to infinite Ramsay's Theorem by defining $f(a_1, \dots, a_n) = 0$ if $(c_{a_1}^{\mathfrak{A}}, \dots, c_{a_n}^{\mathfrak{A}}) \in R^{\mathfrak{A}}$. $\square$

5. As Exercise 4. Just replace the ~~...~~ long sentence φ by sentences φ_r, $r \geq p$, which are as φ except that ~~...~~ instead of p use r and require that $x_1 < c_r$.