Model theory
Exercise 13

1. Notice that if $f: \mathfrak{A} \to \mathfrak{B}$ is not a partial isomorphism then I can show this in finitely many moves in $EF_\omega(\mathfrak{A}, \mathfrak{B})$

2. Proved in lectures.

3. E.g. κ is not a successor (the other case is essentially similar);
Suppose $\kappa = \mu^+$: Let

$T = \{\neg c_i = c_j \mid i < j < \kappa\} \cup \{\varphi\}$ where

$\varphi = \forall x \left(\bigvee_{i < \mu} x = d_i \right)$, where

c_i, $i < \kappa$, and d_i, $i < \mu$, are constants.

4. Notice that G is simple iff

for all $a, b \in G \setminus \{1\}$, there are $n \in \mathbb{N}$ and $c_i \in G$, $i \leq n$, s.t.

$b = (c_0 a_0 c_0^{-1})(c_1 a_1 c_1^{-1}) \cdot \ldots \cdot (c_n a_n c_n^{-1})$

where $a_i \in \{a, a^{-1}\}$ for all $i \leq n$.

5. Let $\mathfrak{A} = (\mathbb{Q}, <)$ and

$\mathfrak{B} = (\omega_1 \times \mathbb{Q}, <^*)$ where

$(\alpha, q) <^* (\beta, r)$ if $\alpha < \beta$ or

$\alpha = \beta$ and $q < r$.

Now $\mathrm{II} \uparrow EF^*(\mathfrak{A}, \mathfrak{B})$ and thus $\mathfrak{A} \equiv_{\infty\omega} \mathfrak{B}$

but in $\mathfrak{A}$ there is no increasing ω_1-sequence

and in $\mathfrak{B}$ there is such.