Model theory
Exercise 12

1. For all T and D, if ~~[illegible]~~ there is $\lambda \geq \kappa$ s.t. T does not have a model of power λ which omits every $p \in D$, then there is $\theta < \beth_{(2^\kappa)^+}$ s.t. there is no model of T of power $\geq \theta$ which omits every $p \in D$ (by 12.8)
The number of such pairs (T, D) is $2^\kappa < cf(\beth_{(2^\kappa)^+})$ from which the claim follows.

2. It is enough to prove the claim for $n=1$. There are 3 possibilities

1° $a \in A$: There are $\leq \omega$ many such types
2° $a \notin A$ and $a > b$ for all $b \in A$. One such type
3° $a \notin A$ and there is $b \in A$ s.t. $a < b$:

Then the type is determined by $\min\{b \in A \mid a < b\}$. $\leq \omega$ possibilities

~~3. Again we may assume that $n=1$ and that $A = [illegible]$ for some [illegible] of power ω. Then for all [illegible] there are $b \in A^n$ and a term t s.t. [illegible]~~

3. By $EM^*(\kappa, \Phi)$ we denote $EM(\kappa, \Phi)$ with the interpretations for the Skolem-functions. Again we may assume that $n=1$. Also we may assume that $A = SH(X)$ for some countable $X \subseteq \kappa$. Then for all $a \in EM(\kappa, \Phi)$ there are an L^s-term t and $b \in \kappa^m$ s.t. $a = t^{EM^*(\kappa, \Phi)}(b)$. Now $t(a/A)$ is determined by t and $t_{at}(b/X)$. Thus the claim follows from Ex. 2.

4. If not, then there is $\mathfrak{A} \models T$ of power κ and countable $A \subseteq \mathfrak{A}$ s.t. $|\{t(a/A) \mid a \in \mathfrak{A}^n\}| \geq \omega_1$. Since T is κ-categorical, $\mathfrak{A} \cong EM(\kappa, \Phi)$. This contradicts Ex 3.