Model theory
Exercise 11
Hints

1. Follows from Lemma 6.1. E.g. T_{acf_0} is s.m.: Since T_{acf_0} has e.q., it is enough to show that the sets definable with atomic formulas are either finite or their complement is finite. This is clear by Lemma 6.1.

2. Let $dom(\mathfrak{A}) = \{(m,n) \in \omega^2 \mid m \leq n\}$ and $(m,n) E^{\mathfrak{A}} (m',n')$ if $n = n'$. Then $Th(\mathfrak{A})$ has e.q. and thus it is easy to see that $\mathfrak{A}$ is minimal. Let $dom(\mathfrak{B}) = \{(m,n) \in (\omega_1)^2 \mid m \leq n\}$ and $(m,n) E^{\mathfrak{B}} (m',n')$ if $n = n'$. E.g. by Theorem 7.8 $\mathfrak{A} \equiv \mathfrak{B}$ and thus by e.q. $\mathfrak{A} \preceq \mathfrak{B}$. Clearly $\mathfrak{B}$ is not minimal.

3. Let $dom(\mathfrak{A}) = \omega \cup (\omega^2)$ and $f^{\mathfrak{A}} : \mathfrak{A} \to \mathfrak{A}$ s.t. $f^{\mathfrak{A}}(n) = n$ and $f^{\mathfrak{A}}((n,m)) = n$. Then $0 \in acl(\{(0,0)\})$ but $(0,0) \notin acl(\{0\})$.

4. "$\Rightarrow$" Clear. "$\Leftarrow$": To see that $a_1 \in \operatorname{acl}^X_{a^*}(A)$ use $\exists x_2 \ldots \exists x_n \, \varphi(X, b, a^*)$.

5. $\varphi(x, c) \in t(a / \Pr(A) \cup a^*)$ if $\varphi(\overline{X}, c)$ is infinite, by Exercise 4.