Model theory
Exercise 10

1. If $0 \in A$, then $\mathrm{aff}(A) = \mathrm{span}(A)$.

Thus by (i) and simple observations it is enough to show that

for all a, b and $A \neq \emptyset$, $b \in \mathrm{aff}(A)$

if $f_a(b) \in \mathrm{aff}(f_a(A))$:

$b \in \mathrm{aff}(A) \iff \exists r_i, a_i$ s.t. $b = \sum r_i a_i$

$\sum r_i = 1$ and $a_i \in A \iff$

$\exists r_i, a_i$ s.t. $\sum r_i = 1$, $a_i \in A$ and

$f_a(b) = (\sum r_i a_i) + a = (\sum r_i a_i) + \sum r_i a$

$= \sum r_i f_a(a_i)$.

2. —

3. Just check the definitions

4. Just check the definitions

5. ~~By~~ Suppose not. By (g2)

there is a finite counterexample.
So we may assume that $(a_i)_{i \in I}$ is
the smalest counterexample and $I = (n+1, <)$
Then there is $i < n$ s.t.
$a_i \in \mathrm{cl}(\{a_j \mid j < n+1, j \neq i\})$. By the

choice of $(a_i)_{i \in I}$, $a_i \notin \mathrm{cl}(\{a_j \mid j < n, j \neq i\})$

by (g3), $a_n \in cl(\{a_j \mid j<n\})$ ↯

6. (i) Just extend a basis of A to a basis of $A \cup B$ ~~[illegible]~~.

(ii) $(\mathbb{R}^{87}, aff)$ is not modular because ~~[illegible]~~ in $\mathbb{R}^{87}$ there are two lines that do not intersect.

$(\mathbb{R}^{87}, aff_{\{a\}})$ is modular because by exercise 1 it is isomorphic with $(\mathbb{R}^{87}, span)$ and a simple exercise in ~~[illegible]~~ linear algebra shows that $(\mathbb{R}^{87}, span)$ is modular. (Keep in mind that span(A) is the ~~[illegible]~~ subspace of $\mathbb{R}^{87}$ generated by A)