

Model theory
Exercise 1
Hints

1. $\forall v_i, (\exp(v_i) = 1 \rightarrow \exp(v_0 \times v_i) = 1)$
2. Let $\psi(x, a)$ be the $L \cup \{R\}$ -formula that defines X . Prove by induction on ψ that there is an L -formula $\psi^*(x, a)$ that also defines X . Notice that the only non-trivial case is the $\psi(x, a) = R(t_1(x, a), \dots, t_m(x, a))$ case.
3. By induction on ψ . The non-trivial case is $\psi = \exists v_i \theta(v_i, x)$. Use g to restrict the quantification to $g(\mathcal{O}^2)$.
4. If $g(x, a)$ defines X , then $g(x, F(a))$ defines $F(X)$. If there are no parameters, the same formula defines both X and $F(X)$ and thus they are the same.

To see that i is not definable without parameters in $(\mathbb{C}, +, \times, 0, 1)$

it is enough to find an automorphism F s.t. $F(i) \neq i$. (Complex conjugation

is such an automorphism. (There was an error in the exercise: with parameters i is definable, $v_0 = i$ defines it.)

5.

