Introduction to the course "Inverse Problems"

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University of Helsinki
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From Effect to Cause. Robustly and Quickly.

Finnish Centre of Excellence in Inverse Problems Research

ACADEMY OF FINLAND

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UNIVERSITY OF HELSINKI

http://wiki.helsinki.fi/display/inverse/Home

Outline

What are inverse problems?

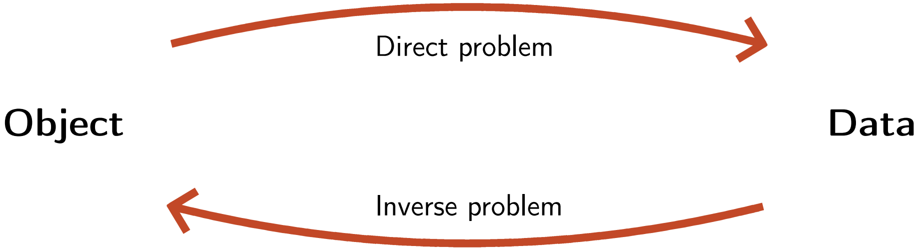
Case: X-ray tomography

Case: glottal inverse filtering

Inverse problems in industry

Practical information about the course

Direct problem: ***given object, determine data***
Inverse problem: ***given noisy data, recover object***

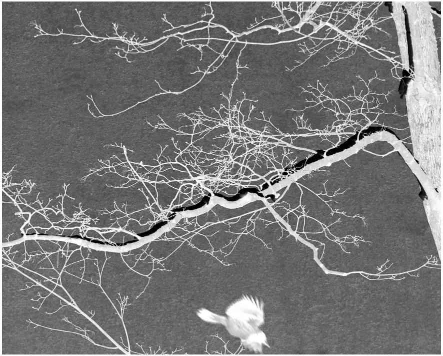


Direct problem: ***given object, determine data***

Inverse problem: ***given noisy data, recover object***

Object (positive photograph)

Data (negative photograph)



Forward map: subtraction from a constant

Direct problem: ***given object, determine data***
Inverse problem: ***given noisy data, recover object***

Object (sharp photograph)

Data (blurred and noisy photo)

Forward map: convolution operator

With appropriate regularization, the blurred image can be sharpened to some extent

Reconstruction

Data (blurred and noisy photo)

With appropriate regularization, the blurred image can be sharpened to some extent

Reconstruction (TV)

Data (blurred and noisy photo)

With appropriate regularization, the blurred image can be sharpened to some extent

Reconstruction (TGV)

Data (blurred and noisy photo)

Thanks to Professor Kristian Bredies for the TGV code

With appropriate regularization, the blurred image can be sharpened to some extent

Reconstruction (TGV)

Original

Thanks to Professor Kristian Bredies for the TGV code

The Hubble space telescope, launched in 1990, first gave blurred images due to a flawed mirror

Hubble telescope

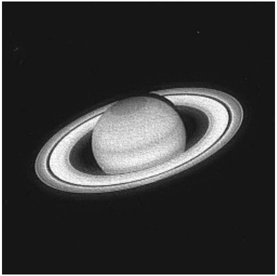
Saturnus (blurred)

Images: NASA, ESA, Quarktet

The mirror flaw was compensated by a deconvolution algorithm

Hubble telescope

Saturnus (blurred)



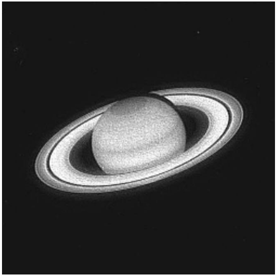
Saturnus (corrected)

Images: NASA, ESA, Quarktet

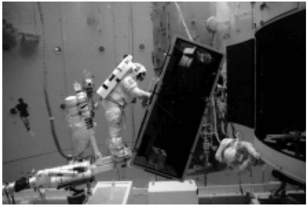
The mirror was replaced in 1993. The new sharp images are further enhanced with deconvolution!

Hubble telescope

Saturnus (blurred)



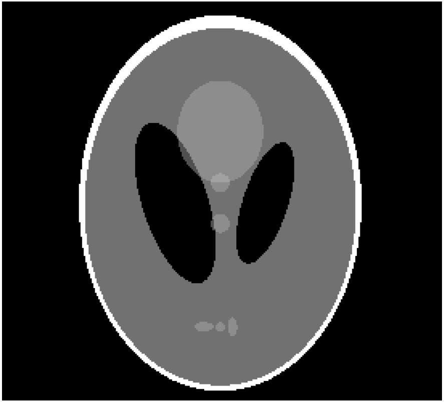
Saturnus (corrected)



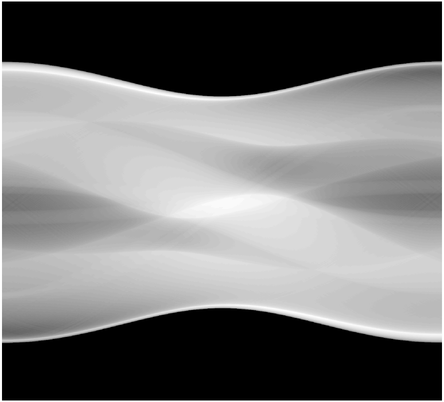
Images: NASA, ESA, Quarktet

Direct problem: ***given object, determine data***
Inverse problem: ***given noisy data, recover object***

Object (X-ray attenuation)



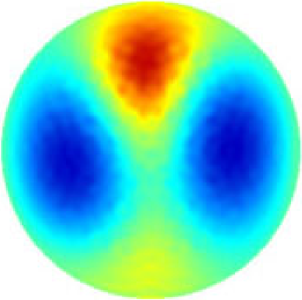
Data (sinogram)

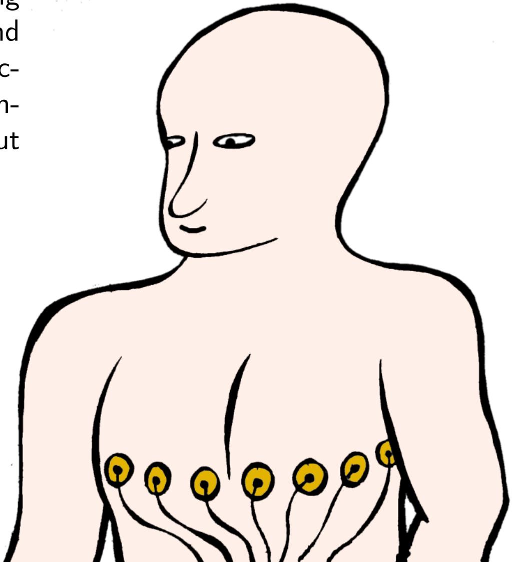


Forward map: discrete Radon transform

In Electrical Impedance Tomography (EIT), currents are fed and voltages measured

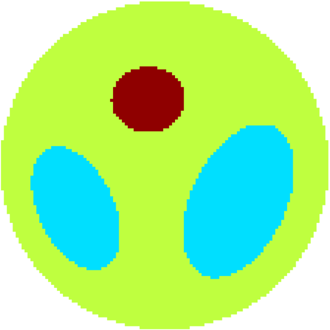
Medical applications: monitoring cardiac activity, lung function, and pulmonary perfusion. Also, electrocardiography (ECG) can be enhanced using knowledge about conductivity distribution.



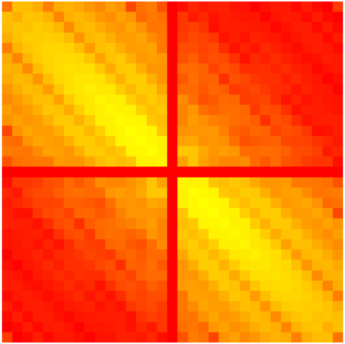


Direct problem: ***given object, determine data***
Inverse problem: ***given noisy data, recover object***

Object (conductivity)



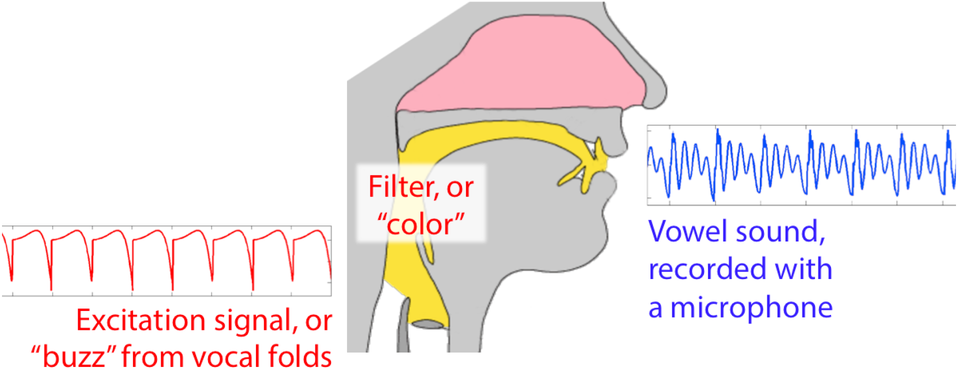
Data (voltage-to-current map)



Forward map: electrical boundary measurements

EIT can perhaps be used for imaging changes in vocal folds due to dehydration

A vowel sound consists of two structural parts: excitation and vocal tract filter



Direct problem: *given object, determine data*
Inverse problem: *given noisy data, recover object*

In glottal inverse filtering (GIF), the data is a vowel sound recorded using a microphone. The aim is to reconstruct the excitation signal and the filter.

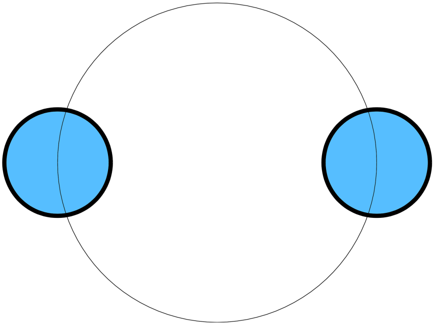
GIF has important applications in

- Computer-generated speech (think Stephen Hawking),
- Speech recognition (think Apple's Siri).

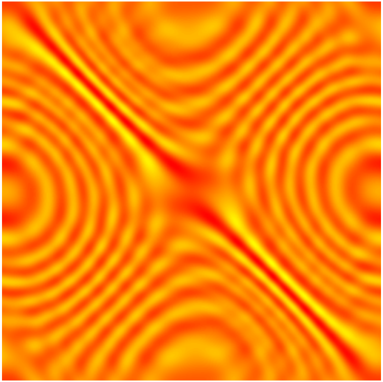
Forward map: bilinear convolution operator

Inverse scattering: send waves through the space and measure effects created by obstacles

Object (sound-hard obstacles)

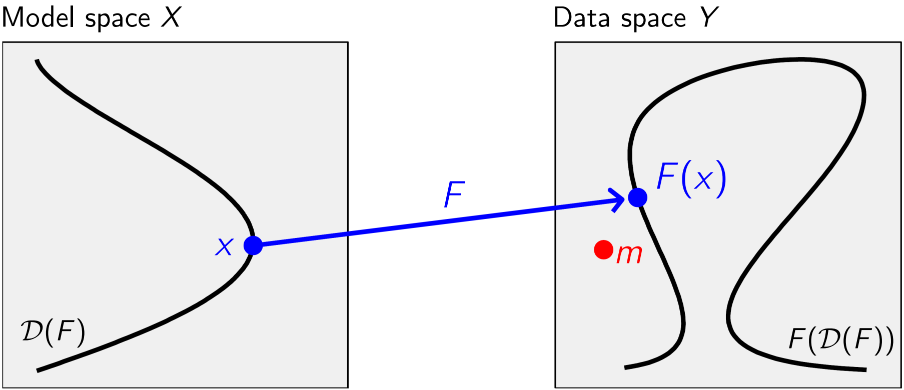


Data (far-field pattern)



Forward map: far-away values of the scattered acoustic wave

Inverse problem = interpretation of an indirect measurement modelled by a forward map F



Consider the measurement model $m = F(x) + \varepsilon$. We want to know x, but all we can do is measure m that depends indirectly on x. Moreover, the measurement is corrupted with noise ε.

Ill-posed inverse problems are defined as opposites of well-posed direct problems

Hadamard (1903): a problem is well-posed if the following conditions hold.

1. A solution exists,
2. The solution is unique,
3. The solution depends continuously on the input.

Well-posed direct problem:
Input x, find infinite-precision data $F(x)$.

Ill-posed inverse problem:
Input noisy data $m = F(x) + \varepsilon$, recover x.

The solution of an inverse problem is a ***set of instructions*** for recovering x stably from m

Those instructions need to be

(i) backed up by rigorous mathematical theory, and

(ii) implementable as an effective computational algorithm.

Ill-posed inverse problems are very sensitive to modelling errors and measurement noise. Therefore, the solution needs *a priori* information about the unknown in addition to measurement data.

In this course we incorporate such *a priori* information using regularization.

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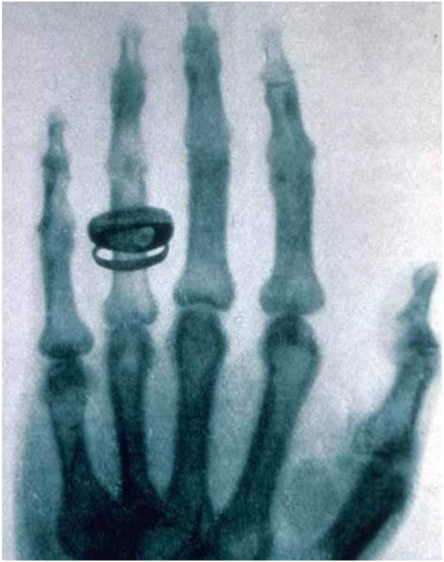
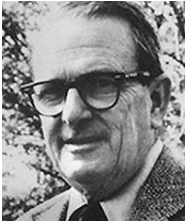
Practical information about the course

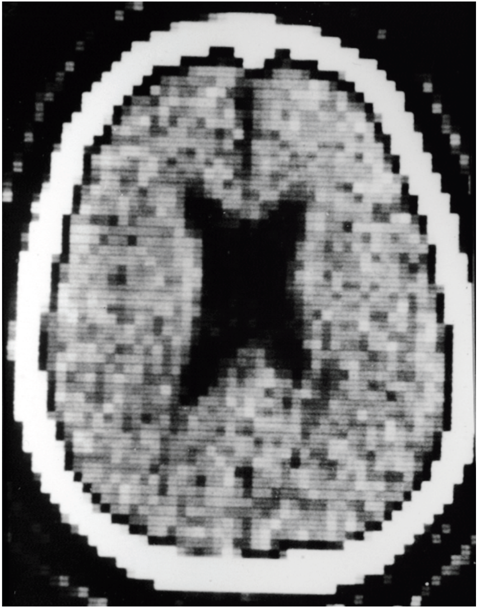
Wilhelm Conrad Röntgen invented X-rays and was awarded the first Nobel Prize in Physics in 1901



Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography



Hounsfield (top) and Cormack received Nobel prizes in 1979.



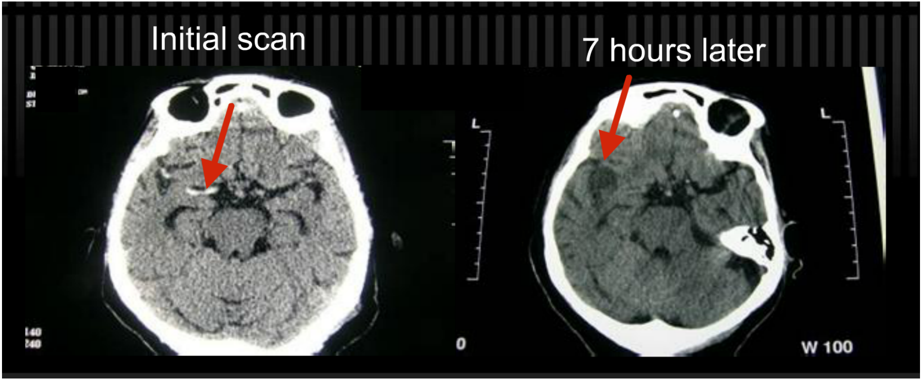
Reconstruction of a function from its line integrals was first invented by Johann Radon in 1917

Johann Radon (1887-1956)

$$f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\overline{F_p}(q)}{q}$$

Contrast-enhanced CT of abdomen, showing liver metastases

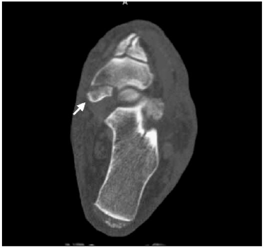
Head CT can be used for detecting and monitoring brain hemorrhage

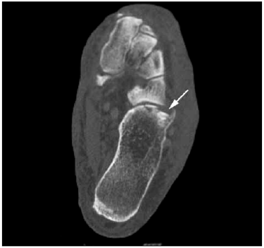


Source: LearningRadiology.com

Unusual variant of the Nutcracker Fracture of the calcaneus and tarsal navicular

Axial slice of the right foot





Another axial slice

Sagittal slice

3D render

[Gajendran, Yoo & Hunter, Radiology Case Reports **3** (2008)]

This sweeping movement is the data collection mode of first-generation CT scanners

https://www.youtube.com/watch?v=TbLaQo3rgEE

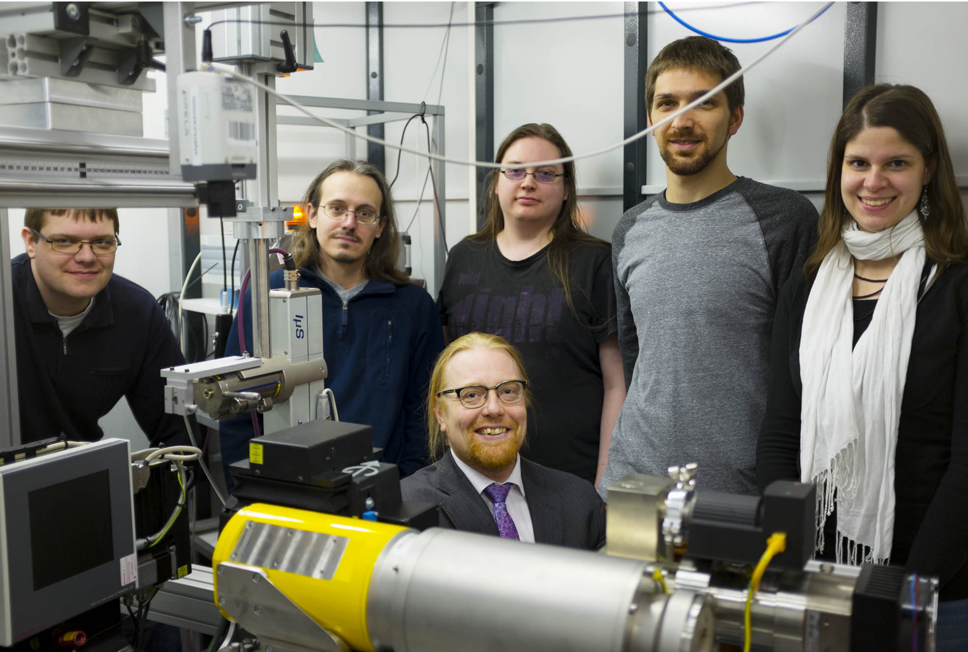
Rotating around the object allows us to form the so-called *sinogram*

https://www.youtube.com/watch?v=5Vyc1TzmNI8

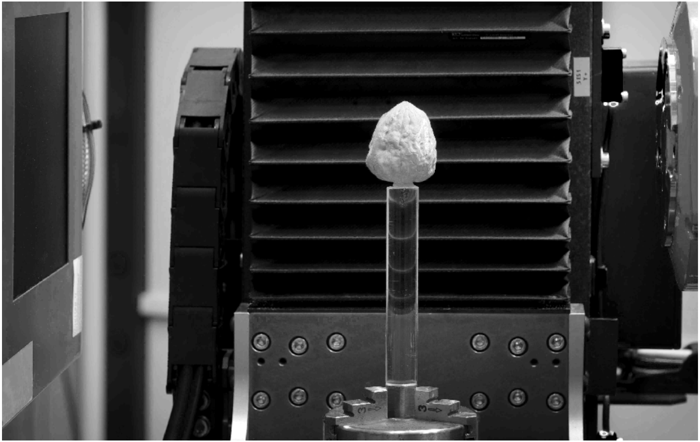
This is an illustration of the standard reconstruction by filtered back-projection

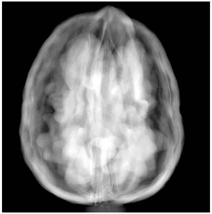
https://www.youtube.com/watch?v=ddZeLNh9aac

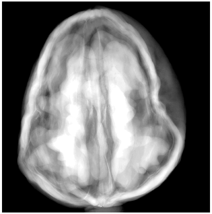
This is Professor Keijo Hämäläinen's X-ray lab



We collected X-ray projection data of a walnut from 1200 directions



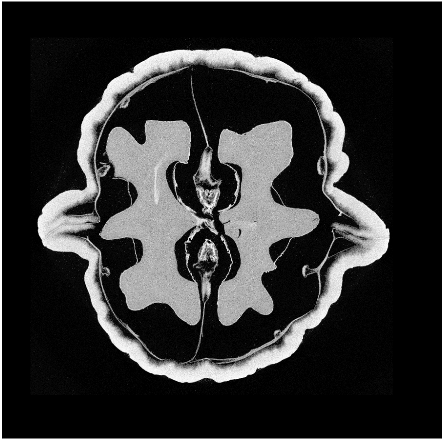




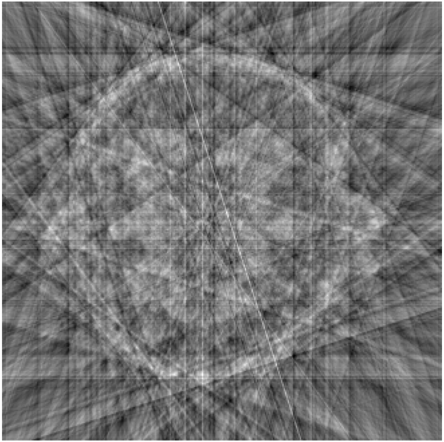
Laboratory and data collection by Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää

Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

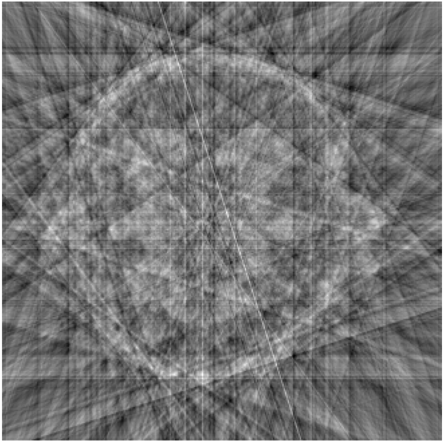


FBP with comprehensive data
(1200 projections)

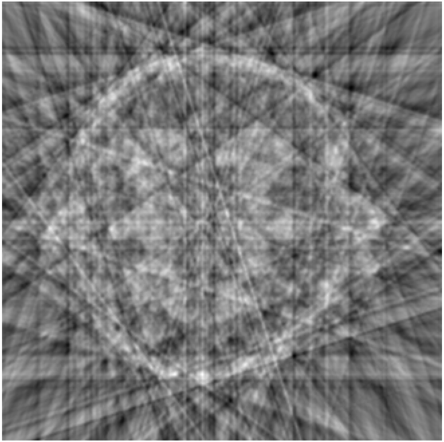


FBP with sparse data
(20 projections)

Sparse-data reconstruction of the walnut using FBP with Hanning filter

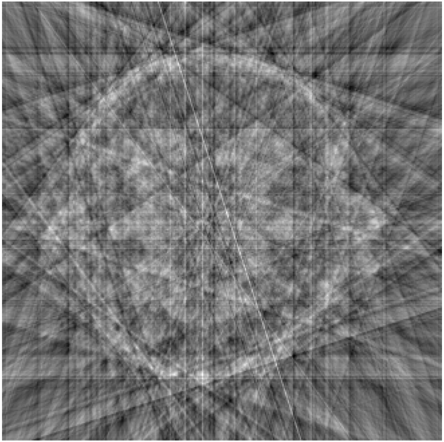


Filtered back-projection

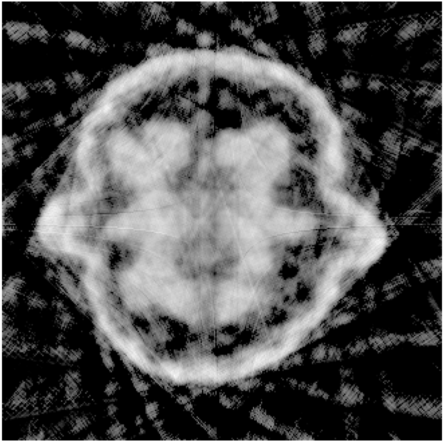


Filtered back-projection, Hanning

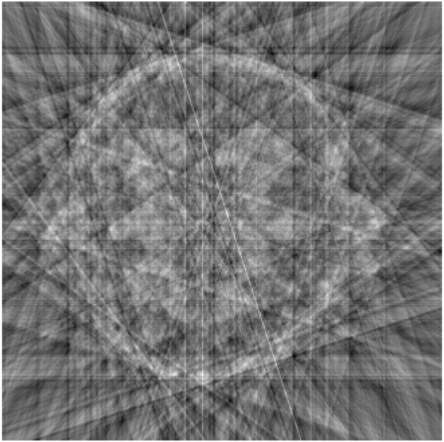
Sparse-data reconstruction of the walnut using non-negative Landweber iteration



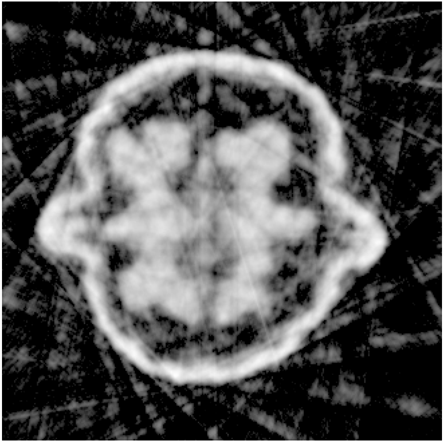
Filtered back-projection



Sparse-data reconstruction of the walnut using non-negative Tikhonov regularization



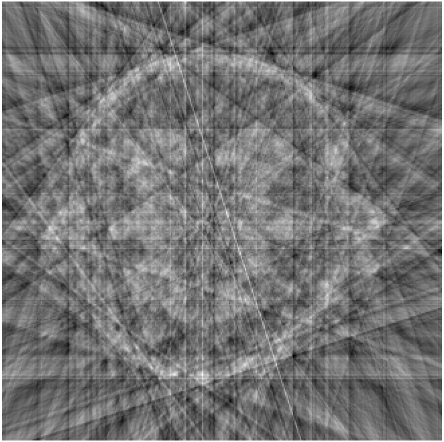
Filtered back-projection



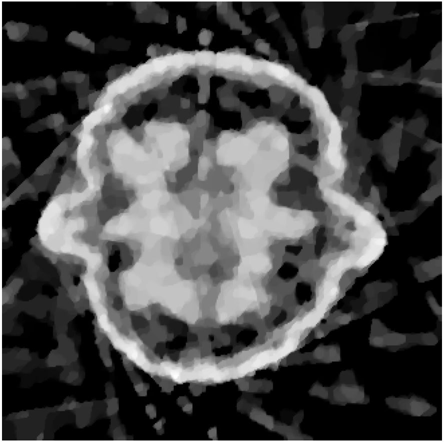
Constrained Tikhonov regularization

$$\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|f\|_2^2 \right\}$$

Sparse-data reconstruction of the walnut using non-negative total variation regularization



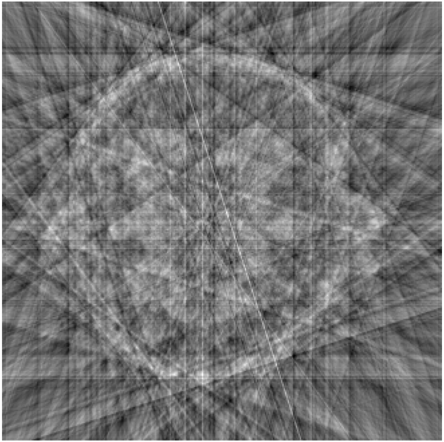
Filtered back-projection



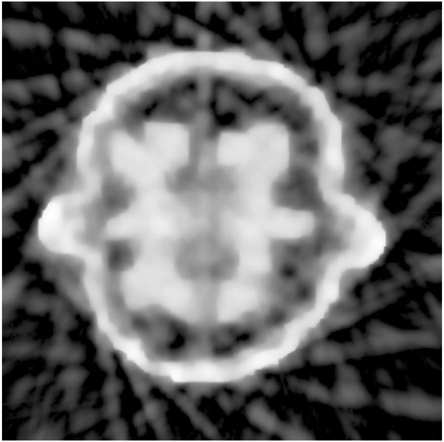
Constrained TV regularization

$$\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}$$

Sparse-data reconstruction of the walnut using non-negative approximate TV regularization



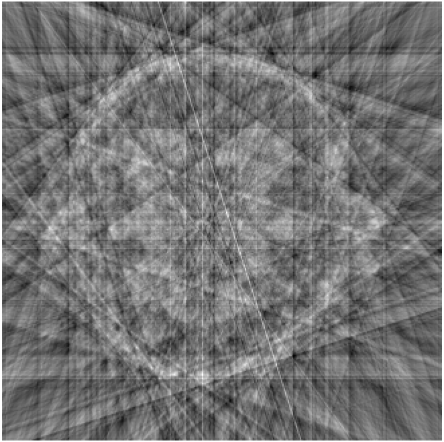
Filtered back-projection



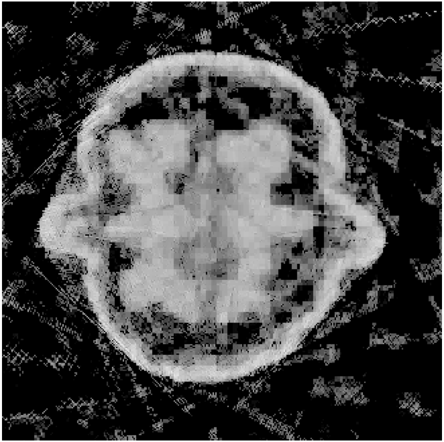
Constrained TV regularization

$$\arg\min_{f \in \mathbb{R}_+^n} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}$$

Sparse-data reconstruction of the walnut using Haar wavelet sparsity



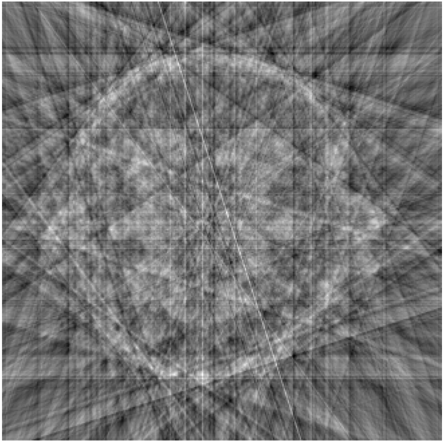
Filtered back-projection



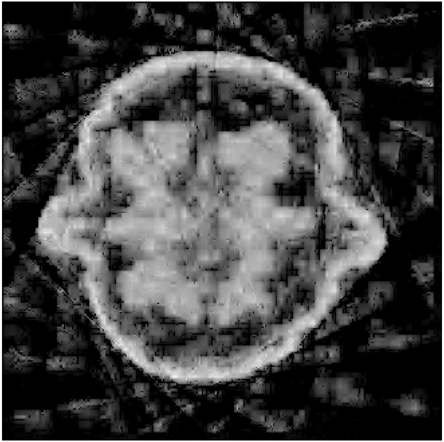
Constrained Besov regularization

$$\arg\min_{f\in\mathbb{R}^n_+}\left\{\|Af-m\|_2^2+\alpha\|f\|_{B^1_{11}}\right\}$$

Sparse-data reconstruction of the walnut using Daubechies 2 wavelet sparsity



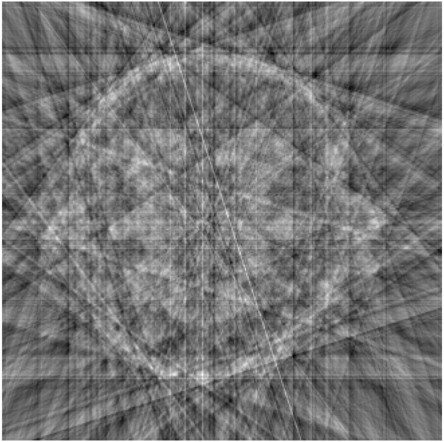
Filtered back-projection



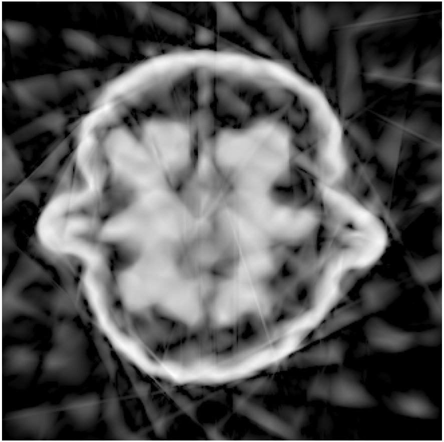
Constrained Besov regularization

$$\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|f\|_{B^1_{11}} \right\}$$

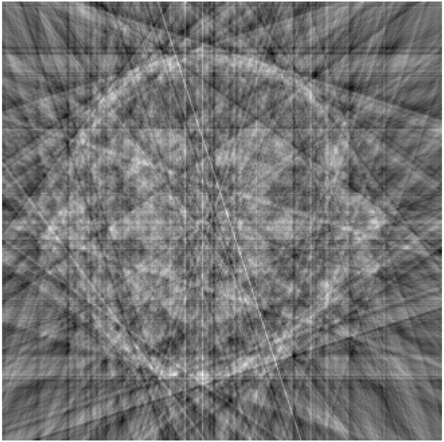
Sparse-data reconstruction of the walnut using shearlet sparsity



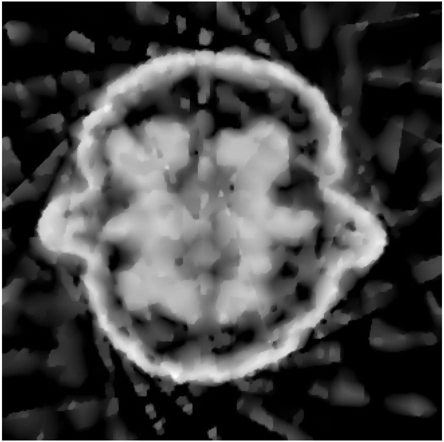
Filtered back-projection



Sparse-data reconstruction of the walnut using Total Generalized Variation (TGV)



Filtered back-projection



TGV: thanks to Kristian Bredies!

The VT device was developed in 2001–2012 by

Lauri Harhanen
Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber

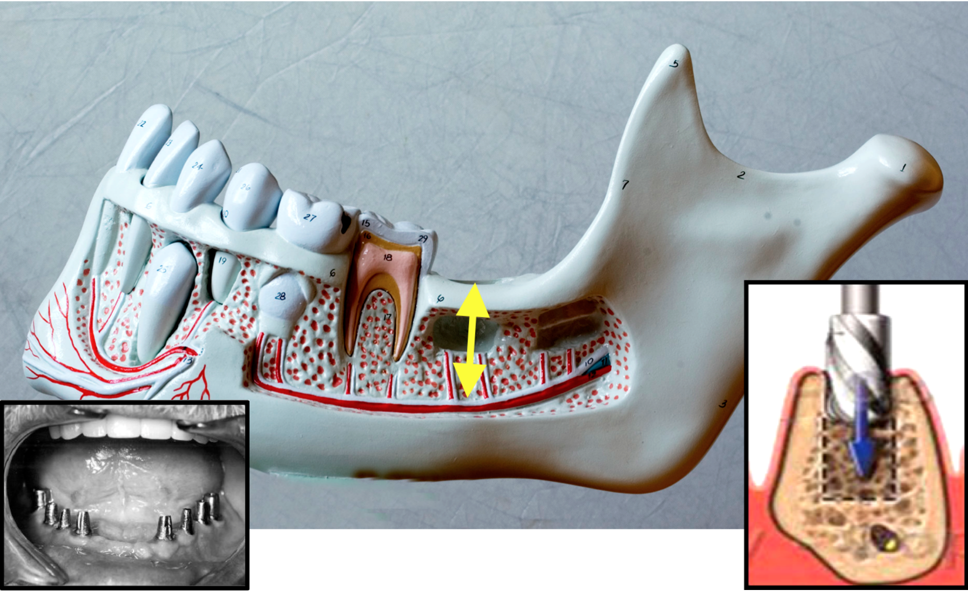
INSTRUMENTARIUM imaging

GE Healthcare

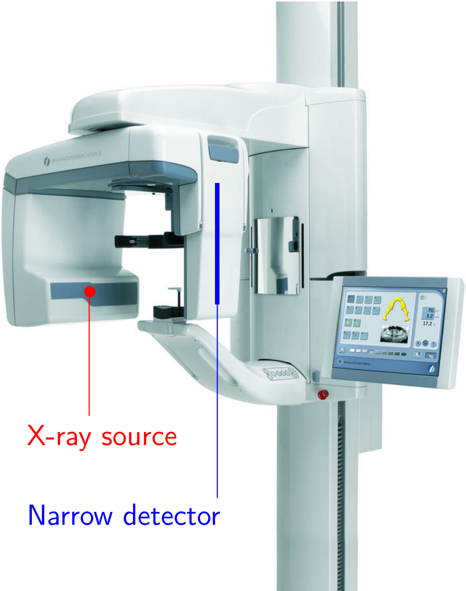
PALODEX GROUP

Tekes

Application: dental implant planning, where a missing tooth is replaced with an implant



Nowadays, a digital panoramic imaging device is standard equipment at dental clinics



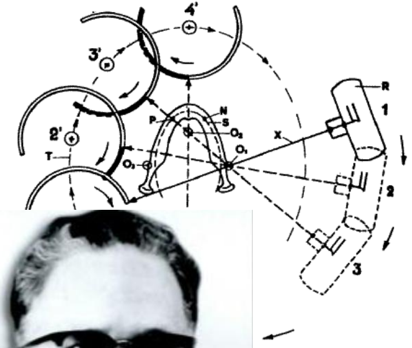
A panoramic dental image offers a general overview showing all teeth and other dento-maxillofacial structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.

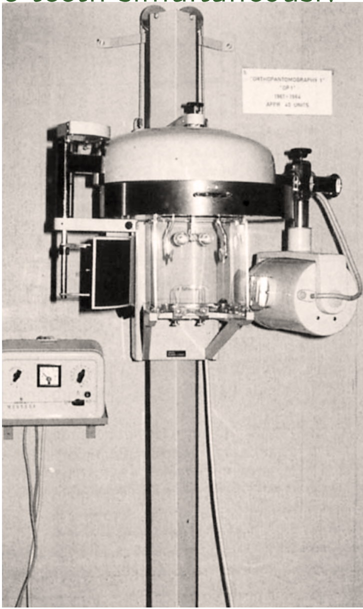
This is the classical imaging procedure of the panoramic X-ray device

https://www.youtube.com/watch?v=QFTXegPxC4U

Panoramic dental imaging shows all the teeth simultaneously



Panoramic imaging was invented by Yrjö Veli Paatero in the 1950's.



We reprogram the panoramic X-ray device so that it collects projection data by scanning

https://www.youtube.com/watch?v=motthjiP8ZQ

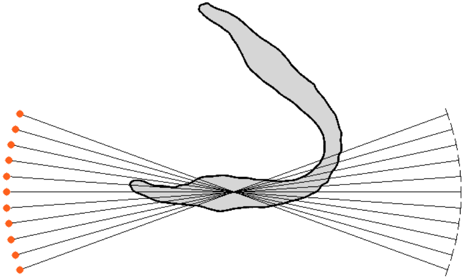
We reprogram the panoramic X-ray device so that it collects projection data by scanning

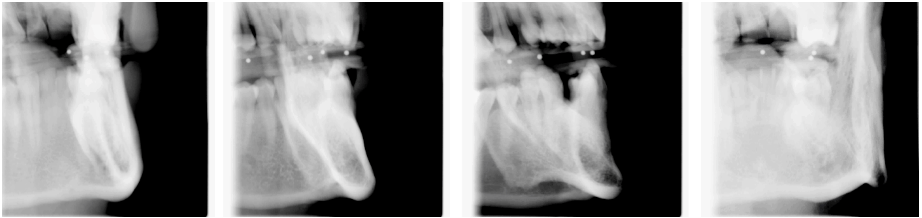
Number of projection images: **11**

Angle of view: **40 degrees**

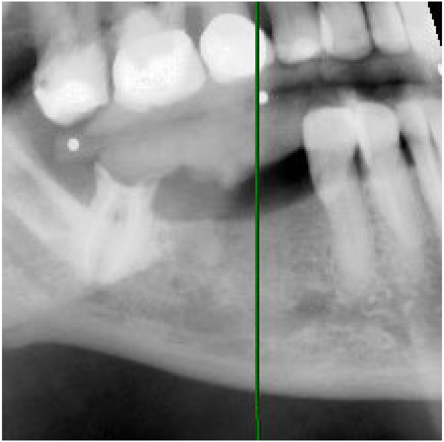
Image size: **1000×1000 pixels**

The unknown vector f has **7 000 000** elements.

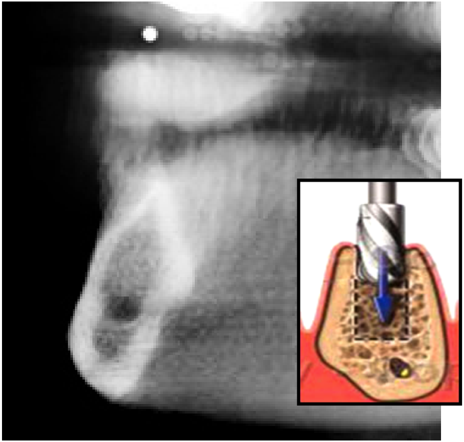




Here are example images of an actual patient: navigation image (left) and desired slice (right).



Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke **2006**, Kolehmainen, Lassas & S **2008**



Cederlund, Kalke & Welander **2009**, Hyvönen, Kalke, Lassas, Setälä & S **2010**, U.S. patent 7269241

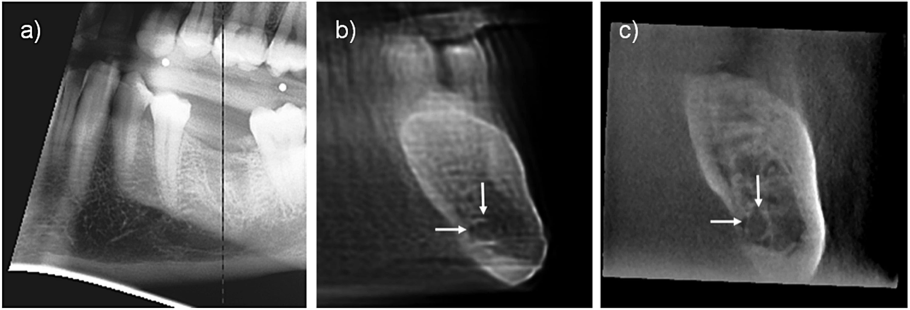
The radiation dose of the VT device is lowest among 3D dental imaging modalities

Modality	μSv
Head CT	2100
CB Mercuray	558
i-Cat	193
NewTom 3G	59
VT device	13

[Ludlow, Davies-Ludlow, Brooks & Howerton **2006**]

The VT device has been available in the international market since 2008.

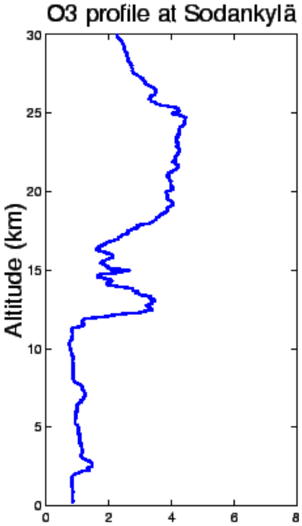
Here the CBCT reconstruction (right) gave 100 times more radiation than VT imaging (middle)



Images from the PhD thesis of Martti Kalke (2015).

The mathematics of X-ray tomography can be used for recovering the ozone layer

European Space Agency
Finnish Meteorological Institute
Envisat and GOMOS projects



Outline

What are inverse problems?

Case: X-ray tomography

Case: glottal inverse filtering

Inverse problems in industry

Practical information about the course

This is a joint work with

Manu Airaksinen, Aalto University, Finland

Paavo Alku, Aalto University, Finland

Harri Auvinen, Numcore Ltd., Finland

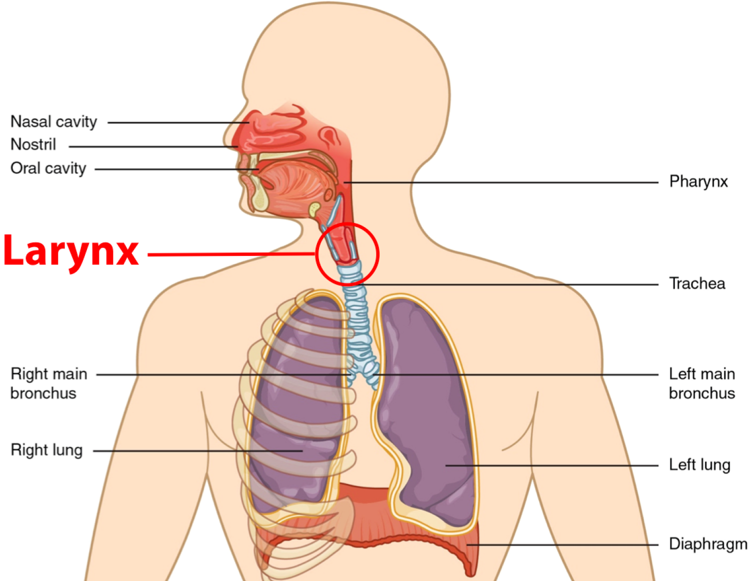
Ismael Rodrigo Bleyer, University of Helsinki, Finland

Lasse Lybeck, University of Helsinki, Finland

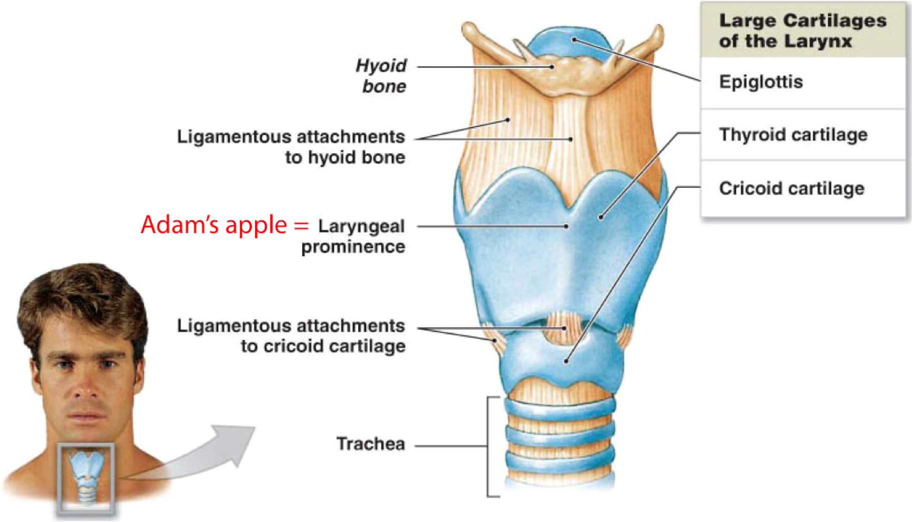
Tuomo Raitio, Apple Corp., California

Brad H. Story, University of Arizona

These parts of the human anatomy are most important for speech production

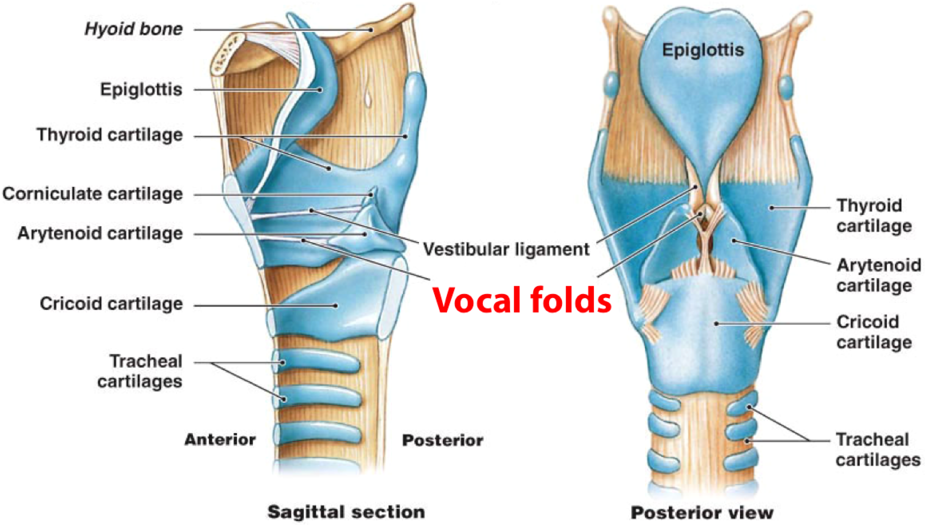


This is frontal view of the larynx



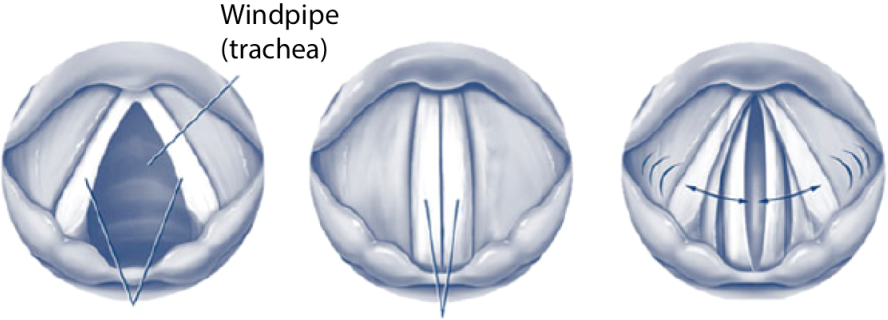
Images: Pearson education

Side and back views of the larynx



Images: Pearson education

The excitation signal at the vocal folds comes from their periodic flapping against each other



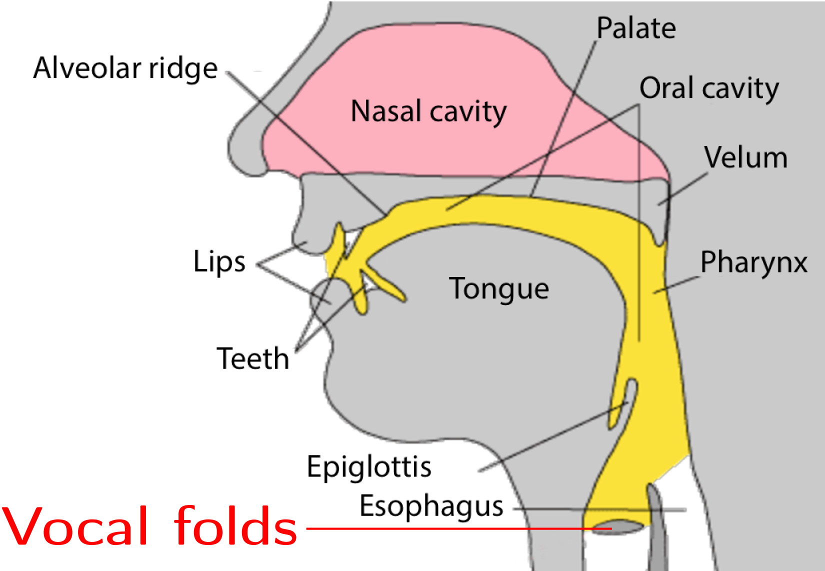
Vocal folds are open when we breathe

Vocal folds are closed when we swallow or lift something heavy

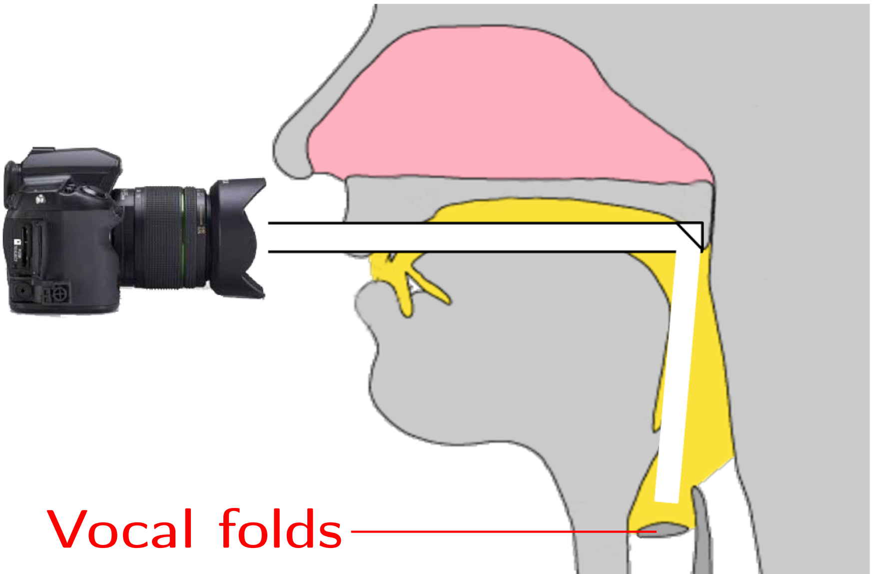
Air causes vocal folds to vibrate between open and closed positions when we talk

Images from www.nidcd.nih.gov

Vocal folds are at the bottom of the vocal tract, which is shown in yellow



If we just had a camera and a mirror, we could see the vocal folds



Meet phoniatrician Ahmed Geneid, PhD

Inserting the video camera (*laryngoscope*) for imaging vocal folds in action

https://www.youtube.com/watch?v=Tw2pYY9g6QM

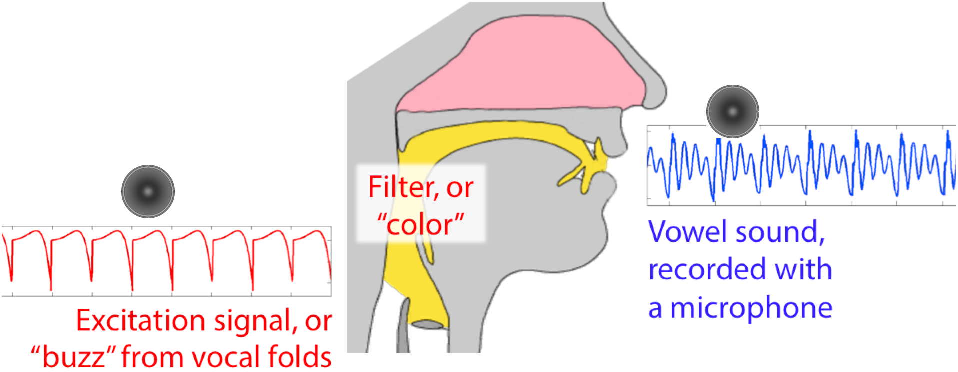
Now we have a top view of the vocal folds!

https://www.youtube.com/watch?v=Tw2pYY9g6QM

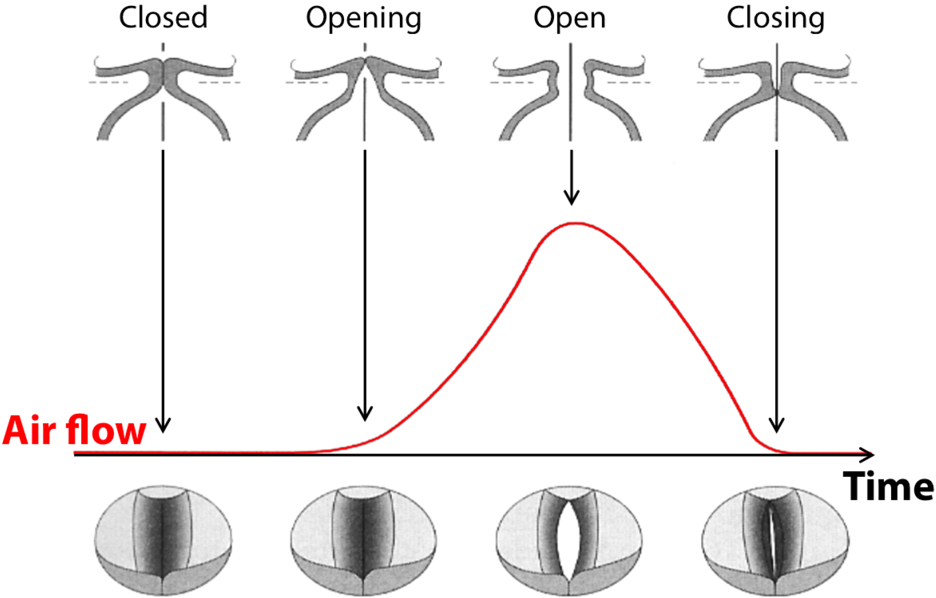
Here you can see the vocal folds moving periodically, creating the excitation signal

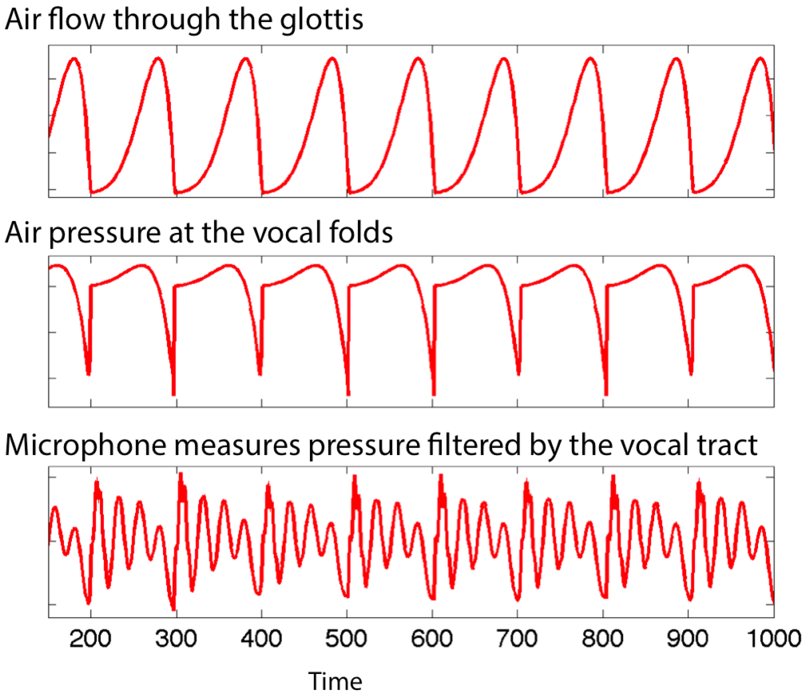
https://www.youtube.com/watch?v=Tw2pYY9g6QM

A vowel sound consists of two structural parts: excitation and vocal tract filter

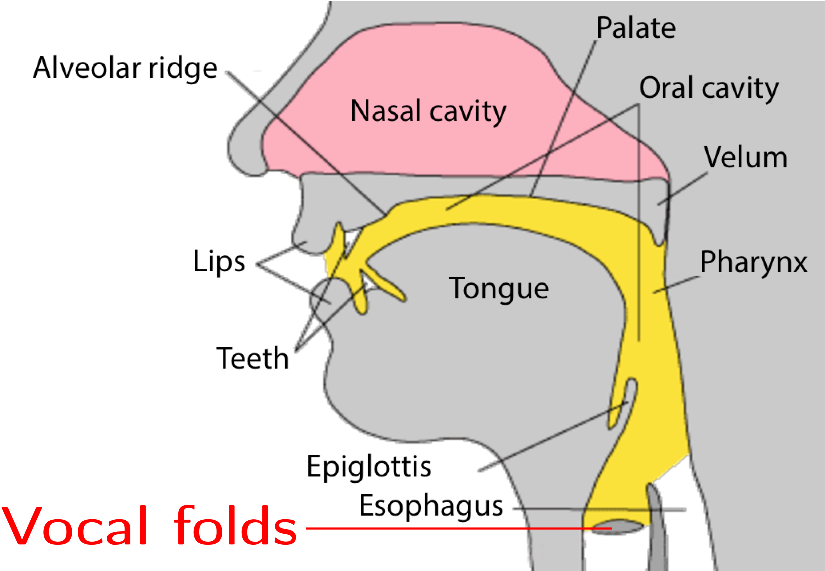


Glottal air flow between the vocal folds

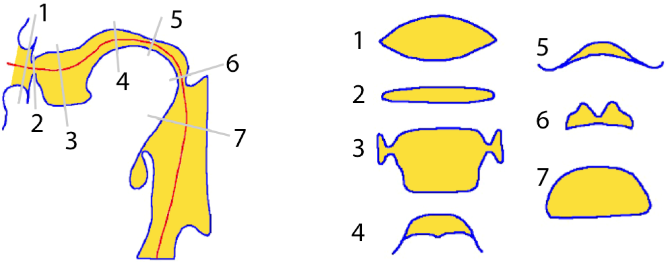


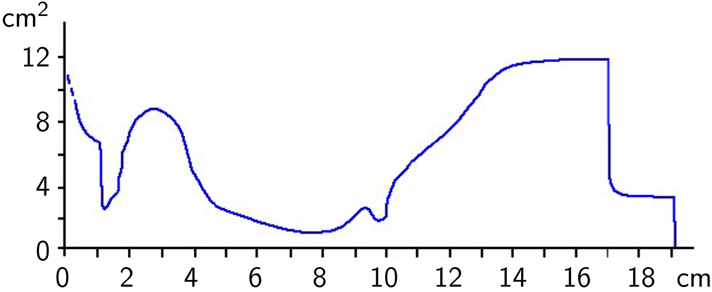

Air flow through the glottis
Air pressure at the vocal folds
Microphone measures pressure filtered by the vocal tract
200
300
400
500
600
700
800
900
1000
Time

Let's take a closer look at the vocal tract (yellow)

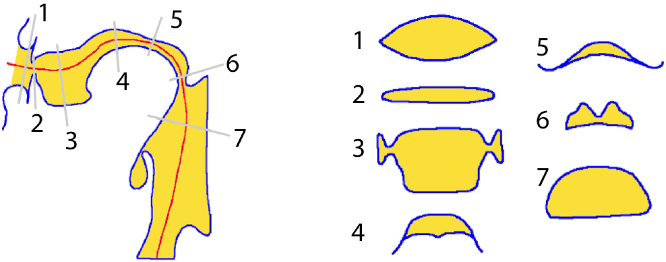


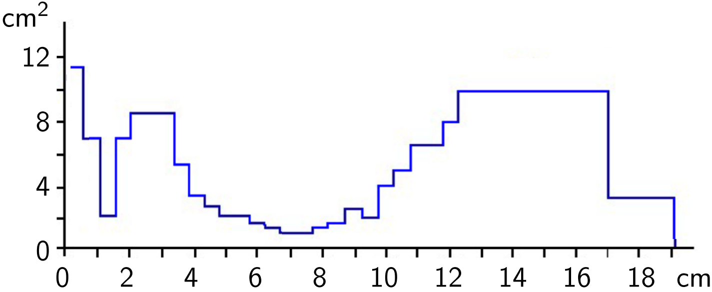
Vocal tract area function shows the size of the tract at different positions along the tube



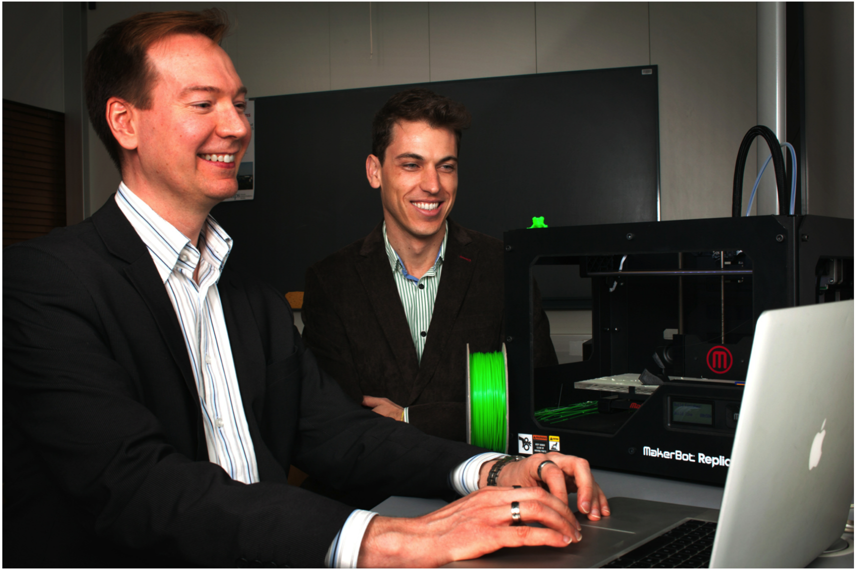


Piecewise constant approximation of the area function gives surprisingly good results

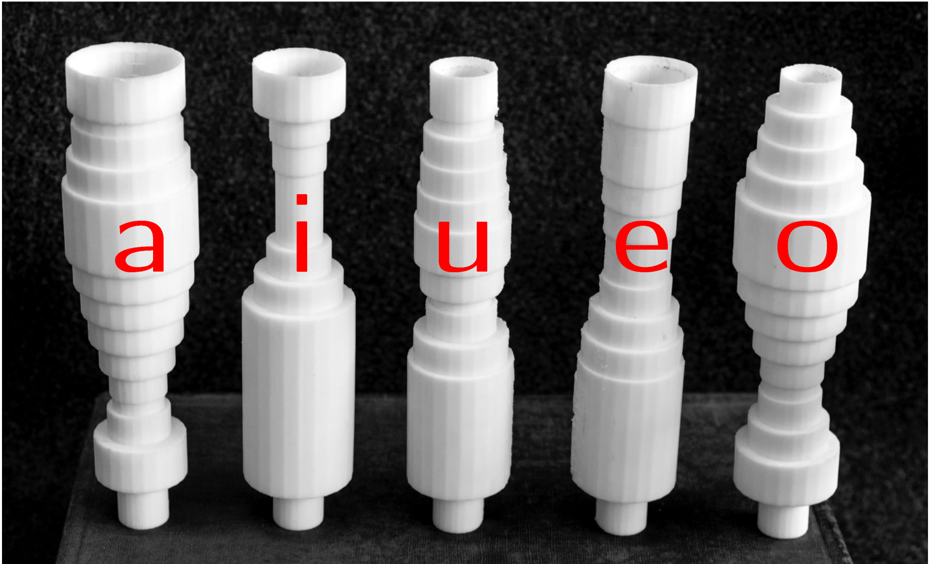




Auvinen and Bleyer at the 3D printer

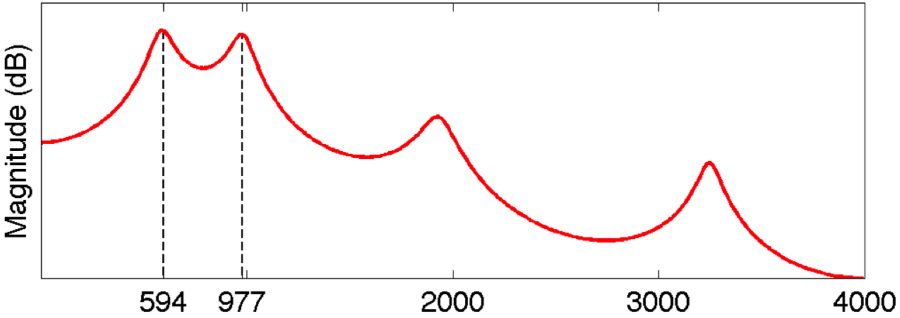


Here are five 3D-printed tube vowel models, adapted from [Arai, Usuki & Murahara 2001]

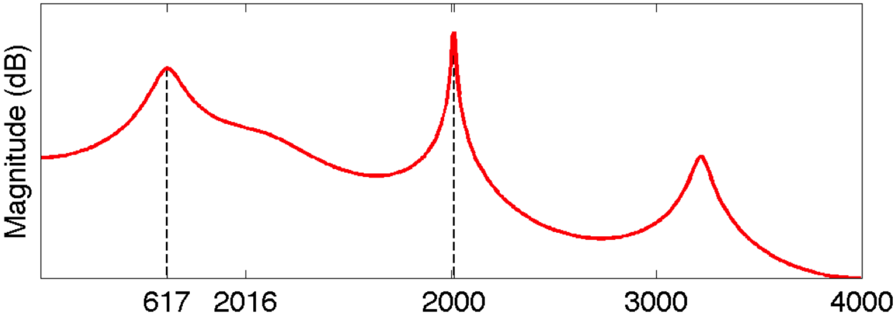


What is a "frequency response"? Let me demonstrate using the rock band AC/DC

The vocal tract filter can be described as a frequency response. Here /a/ (as in “car”)



The vocal tract filter can be described as a frequency response. Here /e/ (as in “element”)



Each vowel has two main resonance frequencies called the first and second formant

	First formant (Hz)	Second formant (Hz)
a	850	1610
i	240	2400
u	250	595
e	390	2300
o	360	640

We define discrete convolution using periodic boundary conditions

Let $p \in \mathbb{R}^n$ and $s \in \mathbb{R}^n$. Convolution $p * s \in \mathbb{R}^n$ is defined by the formula

$$(p * s)_j = \sum_{\ell=1}^{n} p_\ell s_{j-\ell},$$

where $s_{j-\ell}$ is defined using periodic boundary conditions for the cases $j - \ell < 1$ and $j - \ell > n$.

These are the direct and inverse problems related to vowel sounds

Direct problem,
from **cause to effect**:

Given the excitation signal $s(t)$ and the filter $p(t)$, determine the resulting vowel sound signal

$$v(t) = (p * s)(t),$$

where $*$ denotes convolution.

This is a well-posed task, easily implemented as multiplication in the Fourier or $\mathcal{Z}$-transform domain.

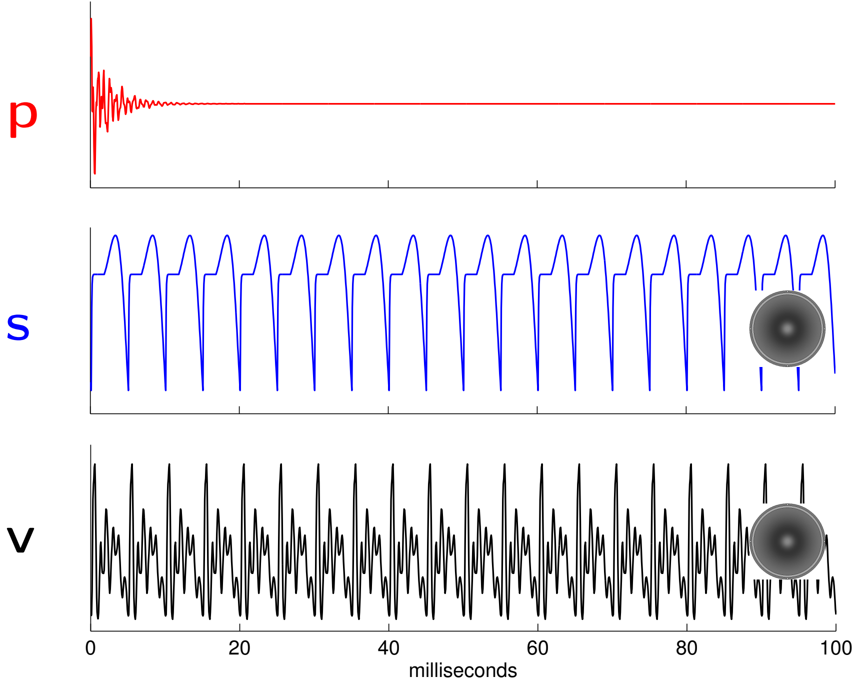
Inverse problem,
from **effect to cause**:

Given a microphone recording $v(t)$ of a vowel sound, use the model

$$v(t) = (p * s)(t) + \varepsilon.$$

Recover the excitation signal $s(t)$ and the filter $p(t)$.

This blind deconvolution problem is called Glottal Inverse Filtering (GIF).


p
s
v
0
20
40
60
80
100
milliseconds

Alternating minimization (AM) for GIF

We follow [**Bleyer** & **Ramlau** 2013, 2015] and look for a minimizing pair $(s, p) \in \mathbb{R}^n \times \mathbb{R}^n$ for

$$\underbrace{\|s * p - v\|^2_{\mathbb{R}^n}}_{\text{discrepancy}} + \underbrace{\tau\|s - s_\varepsilon\|^2_{\mathbb{R}^n} + \beta\|s\|^2_{\mathbb{R}^n} + \alpha\|p\|^2_{\mathbb{R}^n}}_{\text{regularization}}$$

where $\tau > 0$ and $\beta > 0$ and $\alpha > 0$.

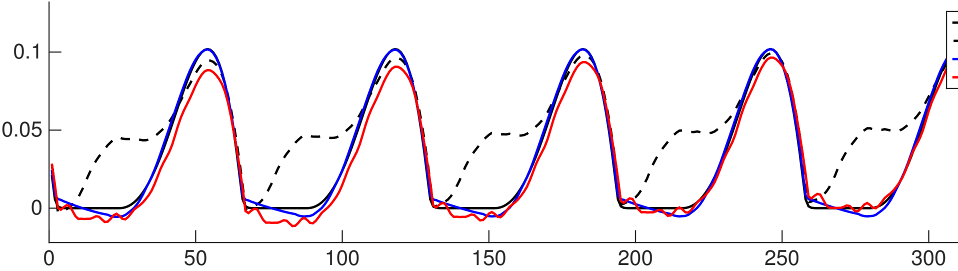
Alternating minimization: Given $s^{(\ell)}$, we compute

$$p^{(\ell+1)} := \underset{p \in \mathbb{R}^n}{\arg\min} \left\{ \|s^{(\ell)} * p - v\|^2_{\mathbb{R}^n} + \alpha\|p\|^2_{\mathbb{R}^n} \right\}.$$

Given $p^{(\ell)}$, we compute

$$s^{(\ell+1)} := \underset{s \in \mathbb{R}^n}{\arg\min} \left\{ \|s * p^{(\ell)} - v\|^2_{\mathbb{R}^n} + \tau\|s - s_\varepsilon\|^2_{\mathbb{R}^n} + \beta\|s\|^2_{\mathbb{R}^n} \right\}.$$

Ilmavirtoja laskettuna eri menetelmillä.



Vokaali: /i/

Äänenkorkeus: $f_0 = 250$ Hz

Auvinen, Raitio, Airaksinen, S, Story & Alku (2014)

Bleyer, Lybeck, Auvinen, Airaksinen, Alku & S (submitted)

The main application of GIF is synthetic speech

Here are examples of high-quality speech signals generated by a computer (it is not a person speaking). GIF plays an important role in such speech synthesis.

http://www.helsinki.fi/speechsciences/synthesis/samples.html

The high-quality samples, developed in **Raitio, Suni, Yamagishi, Pulakka, Nurminen, Vainio** & **Alku** (2010), are based on a Hidden Markov Model (HMM).

Outline

What are inverse problems?

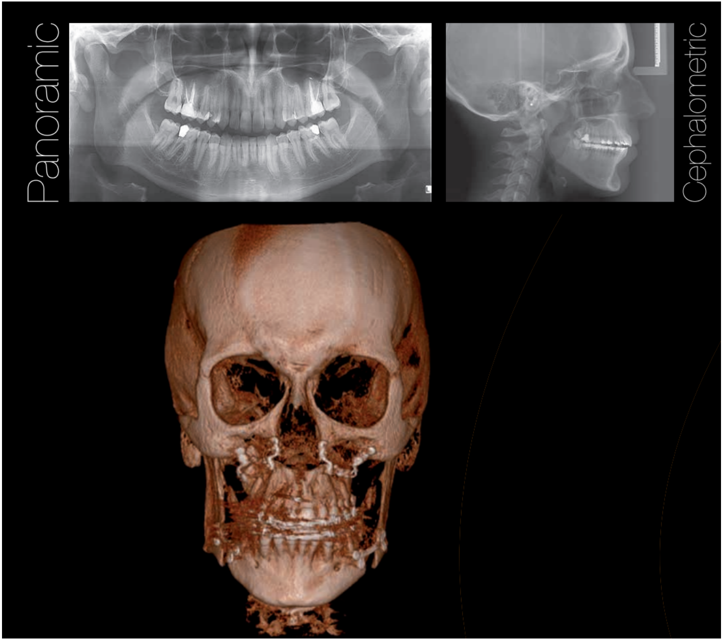
Case: X-ray tomography

Case: glottal inverse filtering

Inverse problems in industry

Practical information about the course

Dental X-ray imaging: www.palodexgroup.com



Medical technology: www.varian.com

Edge™ Radiosurgery System

Using technology designed for radiosurgical ablation, the Edge™ radiosurgery system represents an evolution in the way advanced radiosurgery is delivered.

TrueBeam™ Radiotherapy System

TrueBeam™ system brings leading edge cancer care to communities by positioning clinics at the forefront in the fight against cancer.

VitalBeam™ Radiotherapy System

The VitalBeam™ radiotherapy system is an advanced option for clinics that want sophisticated functionality on a scalable platform.

VARIAN
medical systems

Environmental monitoring: www.vaisala.com

Vaisala Weather Radar WRM200

WRM200 is a dual polarization Doppler weather radar with real time operational hydrometeor classification software.

Vaisala Ceilometer CL51

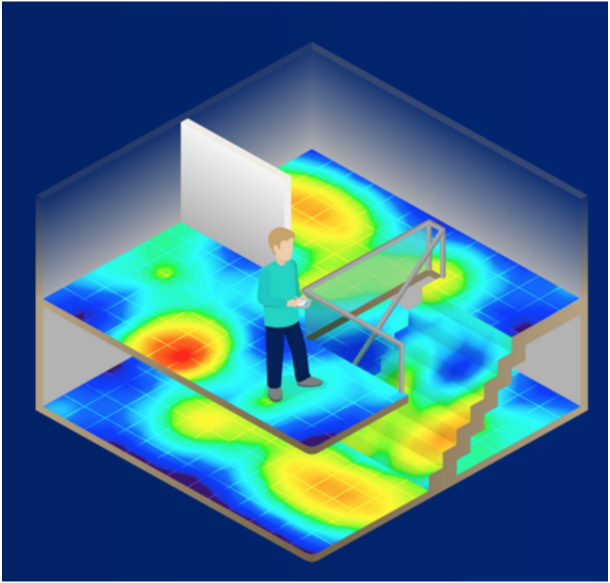
CL51 is designed to measure high range cirrus clouds without unsurpassing the low and middle layer clouds, or vertical visibility in harsh conditions. Cloud reporting range up to 13 km (43,000 ft) and backscatter profiling over full measurement range up to 15 km (49,200 ft). Advanced single-lens design provides excellent performance also at low altitudes.

Vaisala Remote Surface Condition Sensor DSC111

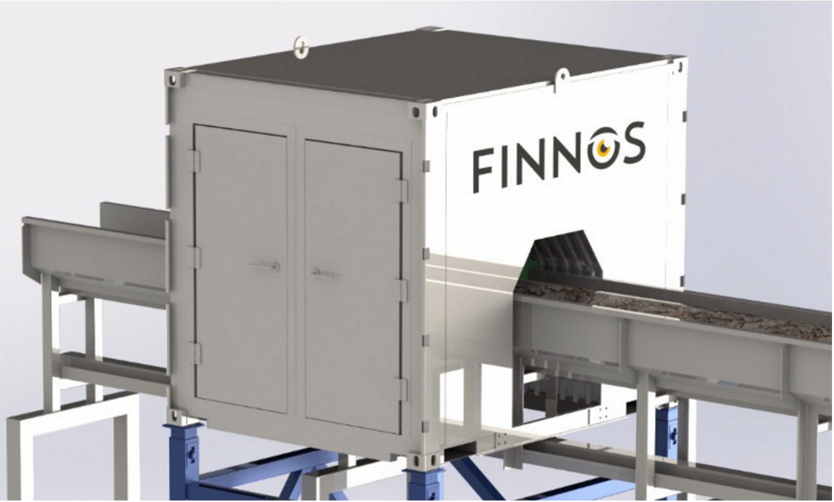
Provides an accurate reading of surface conditions, including water, ice and snow amounts and a high resolution value for grip/friction level. Pole mounted roadside set-up eliminates the disruption caused by slot-cutting, making installation and maintenance relatively easy. When used with a infrared temperature sensor it provides a complete view of the road conditions.

VAISALA

Navigation technology: www.indooratlas.com

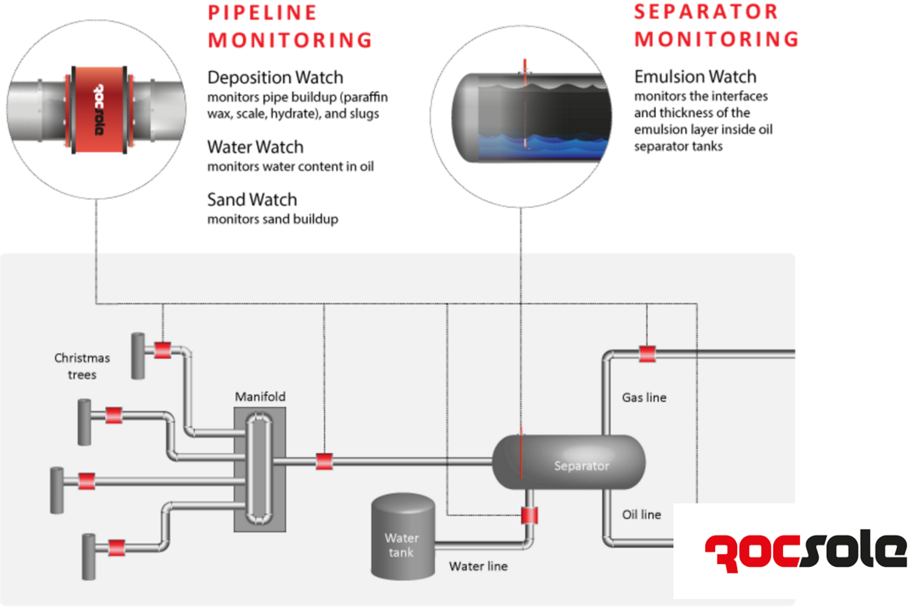


Optimal log cutting: finnos.fi



FINNOS

Process pipeline monitoring: www.rocsole.com



Outline

Goals of the course

1. Learn how to write a practical inverse problem in matrix form: $m = Af + \epsilon$
2. Learn how to detect ill-posedness from a matrix A using Singular Value Decomposition
3. Learn how to overcome ill-posedness by regularization
4. Acquire skills to solve practical inverse problems using Matlab
5. Learn to report your scientific findings in writing

Practical course information

Period III:

Lectures

Wednesday 10-12 (Exactum C123),
Friday 10-12 (Exactum C123).

Exercises

Time and place should be decided

Passing the course (10 sp)

Final exam **and** completing enough exercises.

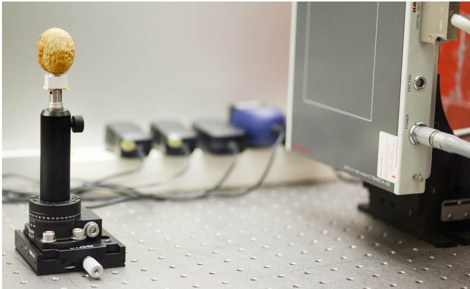
Period IV:

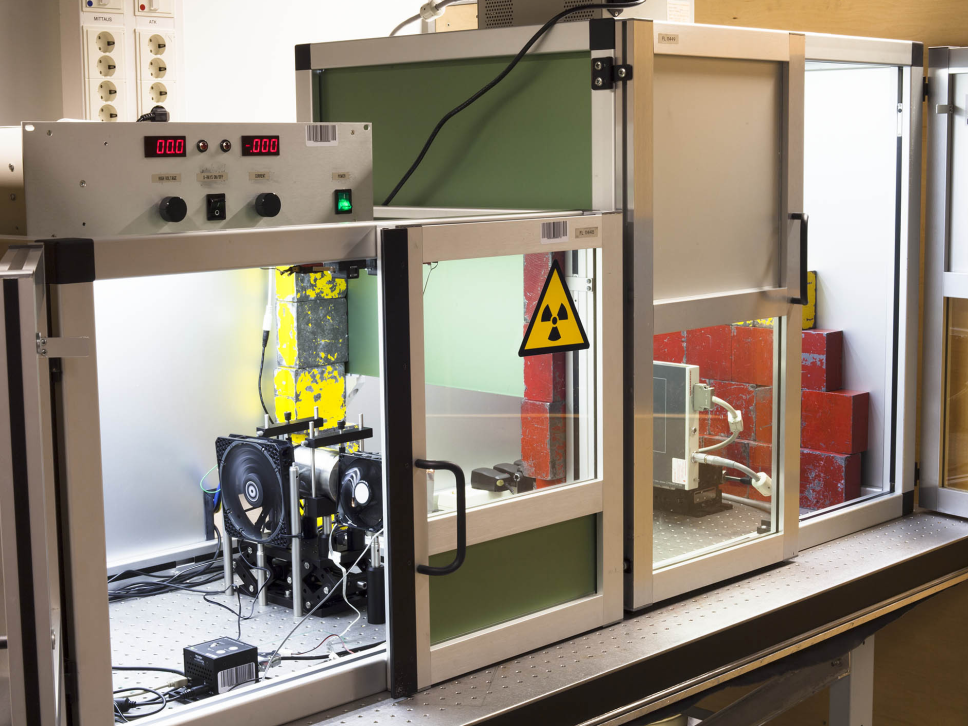
Lectures

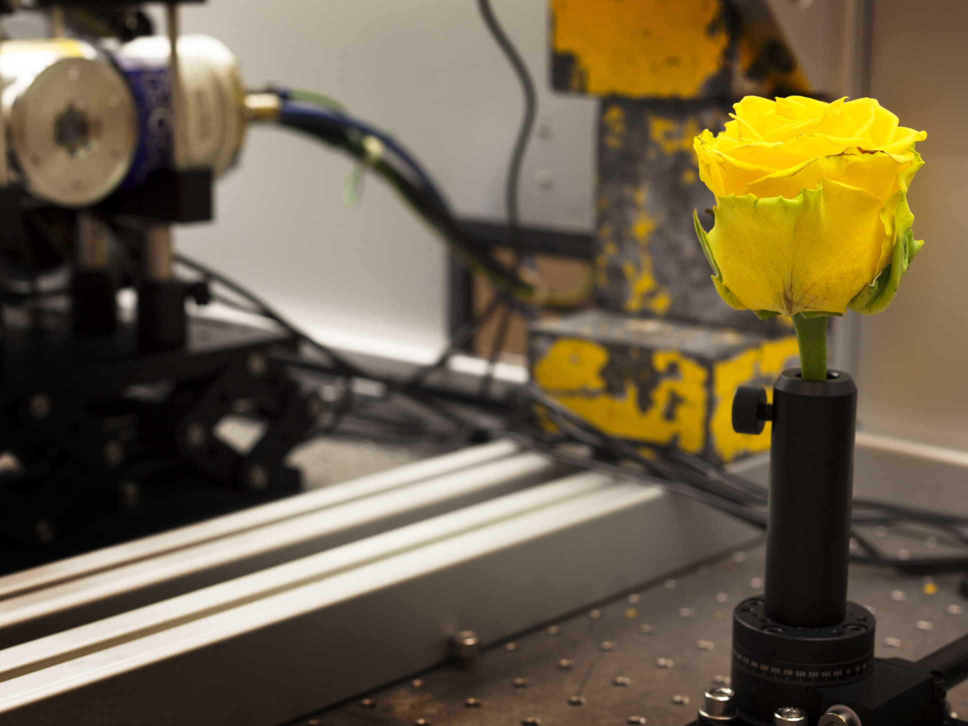
will continue as long as needed.

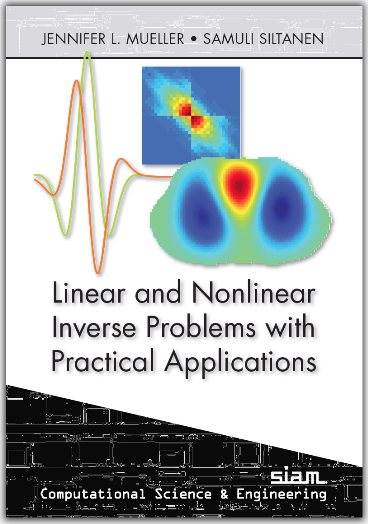
Project work (5 sp)

Computational projet done in pairs. Outcome: poster presentation on a specific day (announced later).




MITTAUS
ATK
00.0
-.000
HIGH VOLTAGE
X-RAYS ON/OFF
CURRENT
POWER





All Matlab codes freely available on a website!

Part I: Linear Inverse Problems

1 Introduction
2 Naïve reconstructions and inverse crimes
3 Ill-Posedness in Inverse Problems
4 Truncated singular value decomposition
5 Tikhonov regularization
6 Total variation regularization
7 Besov space regularization using wavelets
8 Discretization-invariance
9 Practical X-ray tomography with limited data
10 Projects

Part II: Nonlinear Inverse Problems

11 Nonlinear inversion
12 Electrical impedance tomography
13 Simulation of noisy EIT data
14 Complex geometrical optics solutions
15 A regularized D-bar method for direct EIT
16 Other direct solution methods for EIT
17 Projects