GEOMETRIC MEASURE THEORY AND SINGULAR INTEGRALS: HINTS FOR THE EXERCISE SET 3

- Hints for Exercise 1: Take $x \in S$ and notice that you can choose a point $x_0 \in S_0$ and a radius ϵ_0 so that $x \in B(x_0, \epsilon_0)$ and $|T_{\mu,\epsilon_0} 1(x_0)| > \lambda_0$. It is enough (why?) to show that
$$|T_{\mu,\epsilon_0} 1(x) - T_{\mu,\epsilon_0} 1(x_0)| \lesssim 1.$$
Split the function 1 appropriately, and estimate.
- Hints for Exercise 2: baby $T1$, previous exercises.
- Hints for Exercise 3: We did something very similar with the Cauchy transform on the lectures + previous exercises.
- Hints for Exercise 4: This is an "adapted" version of Cotlar's inequality (the weak type boundedness assumption is replaced by the L^1 testing condition). You should follow and modify the proof of Cotlar's inequality appropriately:
  - Fix a cube $Q \subset \mathbb{R}^d$, $\epsilon_0 > 0$ and $x \in Q$ for which you estimate $|T_{\mu,\epsilon_0} 1_Q(x)|$.
  - Move to an appropriate doubling scale $\epsilon > \epsilon_0$ (how does this help with the cube R below?).
  - Notice that the case $\epsilon > c_0 \ell(Q)$ is trivial (how does this help?)
  - Choose an appropriate cube R centred at x so that $B(x, \epsilon) \subset R$.
  - Perform the splitting
  $$\begin{aligned} T_{\mu,\epsilon} 1_Q(x) &= T_{\mu,\epsilon} 1_Q(x) - T_{\mu,\delta}(1_{(2R)^c} 1_Q)(w) \\ &\quad + T_{\mu,\delta} 1_Q(w) - T_{\mu,\delta}(1_{2R} 1_Q)(w), \qquad w \in R. \end{aligned}$$
  Plug this to
  $$T_{\mu,\epsilon} 1_Q(x) = \frac{1}{\mu(R)} \int_R T_{\mu,\epsilon} 1_Q(x) \, d\mu(w).$$
  - Find a way to use the L^1 estimate with R to control the last term. (There is a short way!)