HOMEWORK 6

(1) (20pts) Let $K = \mathbb{Q}(\sqrt{-7})$. A: ring of integers of K.
   (a) Find a $\mathbb{Z}$-basis of A. (hint: use Theorem 1 on page 35 of Book)
   (b) Calculate the absolute discriminant d.
   (c) Calculate the norm bound (round to two decimal places) as in Corollary 1 on page 58 of BOOK.
   (d) Conclude that A is PID.

(2) (20pts) Let $K = \mathbb{Q}(\sqrt{-5})$. A: ring of integers of K.
   (a) Calculate the absolute discriminant d.
   (b) Calculate the norm bound (round to two decimal places) as in Corollary 1 on page 58 of BOOK.
   (c) Show that the norm of the ideal $\alpha = (2, 1 + \sqrt{-5})$ is 2. Show that it is the *unique* ideal in A of norm 2.
   (d) Conclude that the class number $\mathrm{card}(C(A)) = 2$. (Hint: first show that A is not PID, then show its class number is at most 2.)