HOMEWORK 3

(1) (20pts) Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Find $x \in K$ such that $K = \mathbb{Q}[x]$ (i.e., x is a primitive element).

(2) (30pts) Let $A = \mathbb{Z}$, $B = \mathbb{Z}[\sqrt{d}]$ with d a square-free integer.
  (a) Calculate the norm and trace of $x = \sqrt{d}$.
  (b) Calculate the norm and trace of $x = d + \sqrt{d}$.
  (c) Calculate the discriminant of $(\sqrt{d}, d + \sqrt{d})$.

(3) (10pts) Show that $\frac{\sqrt{3}}{2} + \sqrt{2}$ is not integral over $\mathbb{Z}$.

(4) (10pts) Let $A = \mathbb{Z}(\sqrt{d})$ where d is a squarefree negative integer and $d \leq -2$. Show that the only units of A are ± 1.