Computational methods for Integral Equations
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Exercise 1 (March 09, 2017)

1. *Composite quadrature rules*
   Implement the trapezoidal rule (given by the example in the lecture) for computing the integral of a continuous function f. The function should take as input the interval $[a, b]$, the amount of subintervals $n > 0$, and the function itself (as function handle). An initializing script could look like this:

```
expFunc=@(x) exp(x);
a=0; b=1;
n=10;

I=trapzRule(expFunc,[a b],n);
```

   Here *trapzRule* computes the integral. Your result should be $I \approx e - 1$.

   Now instead of using the trapezoidal rule we want to use higher order methods. For that implement Simpson's rule and Boole's rule similarly.

   There are few points to consider: You need to take care here that you apply the quadrature rules to the correct amount of intervals (Simpson's: 2 \ Boole's: 4). Hence n needs to be divisible by $2 \setminus 4$. Also take note that the rules overlap at the outer nodal points and the length of the interval in your weights corresponds to the part your rule is applied, i.e. $2h \setminus 4h$.

2. *Convergence of solutions (Quadrature)*
   We have seen in the lecture that the three quadrature rules we have implemented in the first exercise have convergence rates of different order. Let us check this computationally for the exponential function as above.

   Compute the error for a set of values $n = 4 : 4 : 400$ and record the errors for each quadrature rule. The subinterval length is then $h = 1/n$. You can visualize the error plot as follows (where hh is a vector of interval lengths):

```
loglog(hh,errTrapz), hold on
loglog(hh,errSimps),
loglog(hh,errBoole)
```

   Can you validate the theoretical convergence rates in your plot?

3. *Solving an integral equation: Nyström method*
   Now we can implement the Nyström method. (Given on page 5 of the lecture notes) You can choose any of the quadrature rules from Exercise 1 for computing the integrals.

   Test your algorithm with the following example: Consider the Fredholm integral equation

   $$\lambda x(t) - \int_0^1 e^{st} x(s) ds = y(t),$$

   where $\lambda = 2$, $x(t) = e^t$ and $y(t) = \lambda e^t - \frac{e^{t+1}}{t+1} + \frac{1}{t+1}$.

4. *Convergence of solutions (Nyström method)*
   Repeat the experiment from Exercise 2. Can you see the same behavior?