

Suggestions for topics for an essay in Complex Dynamics, for the second part of the course. Further topics are of course also possible.
Details on these agreed and discussed at lectures, also on Tuesday 21.3, at 10 o'clock at exercise class room.

Litterature:

[B] Beardon, Iteration of Rational Functions, Springer.

[C-G] Carleson-Gamelin. Complex Dynamics. Springer

An explanation: Linearization= conjugation of $P(z)$, in a neighbourhood a fixed point z_0 , by a conformal map to $z \rightarrow az$, where $a = P'(z_0)$.

(The dynamics will depend on argument of $P'(z_0)$, when $|P'(z_0)| = 1$)

Topics:

- Construct a rational map $R(z)$ with empty Fatou set [B], section 11.9, p. 271-
- Linearization = local conjugation at attractive fixed points and at super-attractive fixed points (Böttcher coordinate) [C-G], sections II.2 – II.4
- Analyze dynamics at rational neutral fixed points (several themes in this area, also suitable for theme work) [C-G] section II.5
- Irrational neutral fixed points and examples of non-linearizable cases. [C-G], part of section II.6
- Existence of Siegel discs for polynomials $P(z) = az + z^2$. [requires some further info on boundary values of bounded analytic functions]
- Conformal map onto the complement on the Mandelbrot set, showing e.g. Mandelbrot set is connected. [B], section 9.10
- For $P(z) = z^2 + c$, the c 's with a superattractive periodic point accumulate on the entire boundary of the Mandelbrot set; Misiurewicz -points [C-G], parts of sections VIII.1 and VIII.2
- Expanding rational maps and Lebesgue measure of their Julia sets, subhyperbolic rational maps. [C-G], parts of section V.2, V.3.

- The Euler characteristic and Riemann-Hurwitz formula for proper analytic mappings in domains. [B] sections 5.3-5.4
- The symmetry groups of the Julia sets. [B], part of section 9.5
- Analyze, describe and make a program drawing Julia sets and the Mandelbrot set.
- Dynamics of (general) analytic functions in the unit disc; Wolff-Denjoy theorem and dynamics of Möbius transforms. [C-G], section IV.3, further literature for Möbius maps.