WEIGHTED INEQUALITIES FOR MULTILINEAR SINGULAR INTEGRALS
SET 1

You can pass the course by completing the exercises and returning your written solutions. Return written solution of this set of problems directly to Wendolín Damián during lectures, by email or at office B410. **Deadline: Monday, April 11.**

In the following we are going to consider $m+1$ numbers p together with $1 \leq p_1, \cdots, p_m < \infty$ verifying $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_m}$. We are going to use $\vec{P} = (p_1, \ldots, p_m)$ and $\vec{w} = (w_1, \ldots, w_m)$, to denote a vector of indexes verifying the previous condition and weights, respectively.

EXERCISE 1. Prove that if $\vec{w} \in A_{\vec{P}}$, then the weight

$$\nu_{\vec{w}} = \prod_{j=1}^{m} w_j^{p/p_j}.$$

belongs to A_{mp}.

EXERCISE 2. Prove that if every $w_j \in A_{p_j}$, then

$$\prod_{j=1}^{m} A_{p_j} \subsetneq A_{\vec{P}}.$$

Hint: Find $\vec{w} = (w_1, w_2) \in A_{\vec{P}}$ such that w_j, $j = 1, 2$ are not locally integrable.

EXERCISE 3. Prove that if $\vec{w} \in A_{\vec{P}}$, then

$$[\sigma_j]_{A_\infty} \leq C[\vec{w}]_{A_{\vec{P}}}^{p_j'/p},$$

where $\sigma_j = w_j^{1-p_j'}$ and $[\cdot]_{A_\infty}$ denotes the Fujii-Wilson A_∞ constant.
Hint: Use the proof of Proposition 5.3 in the lecture notes.

EXERCISE 4. Prove that Theorem 6.1. is sharp.
Hint: Let $0 < \varepsilon < 1$ and use the following weights and functions

$$w_i(x) = |x|^{(1-\varepsilon)(p_i - 1)}$$

$$f_i(x) = x^{-1+\varepsilon} \chi_{(0,1)}(x), \quad i = 1, \ldots, m.$$

EXERCISE 5. Let $\mathcal{A}_{\mathcal{D},\mathcal{S}}$ be a multilinear sparse operator and assume that $p_1 = p_2 = \ldots = p_m = m + 1$ and $f_i \in L^{p_i}(w_i)$. Prove that

$$||\mathcal{A}_{\mathcal{D},\mathcal{S}}(\vec{f})||_{L^p(\nu_{\vec{w}})} \lesssim [\vec{w}]_{A_{\vec{P}}} \prod_{i=1}^{m} ||f_i||_{L^p(w_i)}.$$

Hint: Use duality to define the $L^p(\nu_{\vec{w}})$ norm.