Forcing

Exercise 5

1. $p \perp q$ for all $q \in A$ iff there is no $r \geq p$ s.t. $r \in D$

2. By (i) $G \cap D \neq \emptyset \Rightarrow G \cap A \neq \emptyset$. Since all elements of G are compatible, $G \cap A$ contains at most one elem.

3. Let p be such that $p \Vdash$ "$\dot{f}$ is a function $\hat{\theta} \to \hat{\delta}$" and $\tau = \{ (\delta, q) \mid q \leq p \wedge \exists r\, ((\delta, r) \in \dot{f} \wedge q \leq r) \} \cup$

$$\{ (\widehat{(\alpha, 0)}, q) \mid \alpha < \theta \wedge q \perp p \}$$

4 E.g. $\underset{\alpha<\kappa}{\Delta} C_\alpha$ is closed: Suppose $\gamma_i \in \underset{\alpha<\kappa}{\Delta} C_\alpha$, $i < \gamma$ are such that $\gamma_i < \gamma_j$ if $i < j$. and let $\delta = \bigcup_{i<\gamma} \gamma_i$ and $\beta < \delta$. We need to show that $\delta \in C_\beta$. Let $i < \gamma$ be such that $\beta < \gamma_i$. Then $\gamma_i \in C_\beta$. Thus $\delta \in C_\beta$ since C_β is closed. and $\delta = \bigcup \{ \gamma_i \mid i < \gamma, \gamma_i > \beta \}$

5. Easy

6. If not, there are $(\alpha, n) \in \kappa \times \omega$ and $p \in G$ s.t. $p \Vdash (\hat{\alpha}, \hat{n}) \notin \mathrm{dom}(\bigcup \dot{G})$

Let $q = p \cup \{ ((\alpha, n), 0) \}$. Then $q \Vdash (\hat{\alpha}, \hat{n}) \in \mathrm{dom}(\bigcup \dot{G})$

Since $q \leq p$ we have a contraduction.