Forcing

Exercise 3

1. (ii) e.g. ("$<$ is transitive")^V iff

$\forall x \in V\ \forall y \in V\ \forall z \in V\ ((x \in^V a \wedge y \in^V a \wedge z \in^V a \wedge x <^V y \wedge y <^V x) \to x <^V z)$ iff $\forall x, y, z \in a\ ((x < y \wedge y < z) \to x < z)$

iff "$<$ is transitive" ($x <^V y$ means $(x, y) \in^V <$)

(iv) Show that $\omega \in V$ and that for all $\alpha \in On^V$, "α is successor"^{V} $\Leftrightarrow$ α is a successor

(v) Show that for all $n \in \omega$, $V_n^V = V_n$ (by induction)

and notice that for this it is enough

to show that $V_n \subseteq V_n^V$.

2. Enumerate all dense subsets of P: D_n, $n < \omega$, (V is countable) and choose p_i, $i < \omega$, so that $p_0 = p$ and $p_{i+1} \leq p_i$ and $p_{i+1} \in D_i$.

Let $G = \{q \in P \mid q \geq p_i$ for some $i < \omega\}$.

3. $D = C \cup \{q \in P \mid q \perp p\}$ is dense.

4. Apply Th. 1.2.3 to the following classfunction $G: V \to V$:

$$G(x) = \begin{cases} 1 & \text{if } (*) \text{ below holds} \\ 0 & \text{o/w} \end{cases}$$

$(*)$ for some a, $x : TC(a) \to 2$ and for all $b \in a$ there are c and $p \in P$ s.t. $b = (c, p)$ and $x(c) = 1$.

5. $\hat{a}_G = \{ \hat{b}_G \mid b \in a \} \overset{\text{ind. ass}}{=} \{ b \mid b \in a \} = a$

6. (i) every G that contains q contains p.

(ii) $V[G] \models \varphi \wedge \psi$ iff $V[G] \models \varphi$ and $V[G] \models \psi$

(iii) $\vdash \varphi \to \psi$ implies that $V[G] \models \varphi \to \psi$ because ZFC proves soundness.