Department of Mathematics and Statistics, University of Helsinki
Numerical methods and the C language, Winter and Spring 2016

Workshop 11

Exercise 11 FILE: h11.tex printed on April 11, 2016at 12.51.

1. A bounded optimization problem can sometimes be handled by change of variables in the following way. Assume that the task is to minimize the Rosenbrock function

$$f(x,y) = 100(x*x-y)^2 + (1-x)^2$$

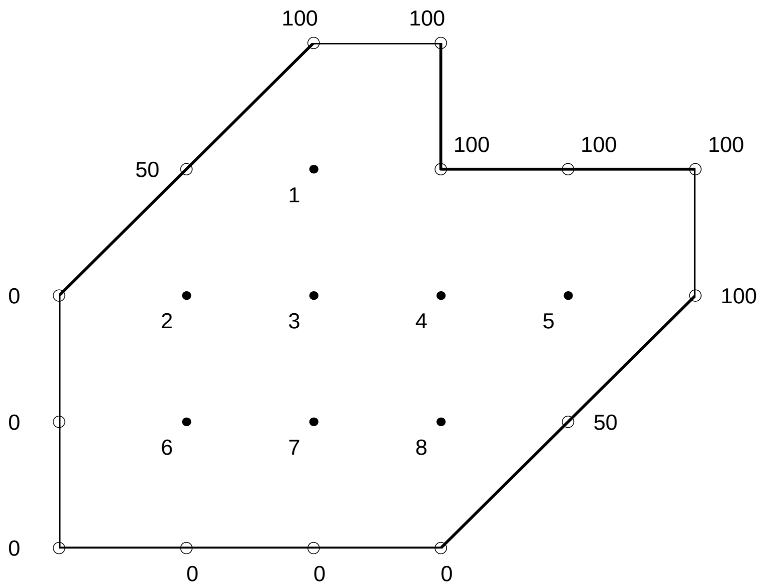
with constraints $x \geq -2$ and $y \geq 2$. We define new variables u and v such that $x = -2 + u^2, y = 2 + v^2$, when the new target function gets the form

$$g(u,v) = 100((-2+u^2)^2 - 2 - v^2)^2 + (1+2-u^2)^2.$$

This is then minimized without constraints, and from its solution u_0, v_0 we obtain the solution to the original problem, $x_0 = -2 + (u_0)^2, y_0 = 2 + (v_0)^2$.

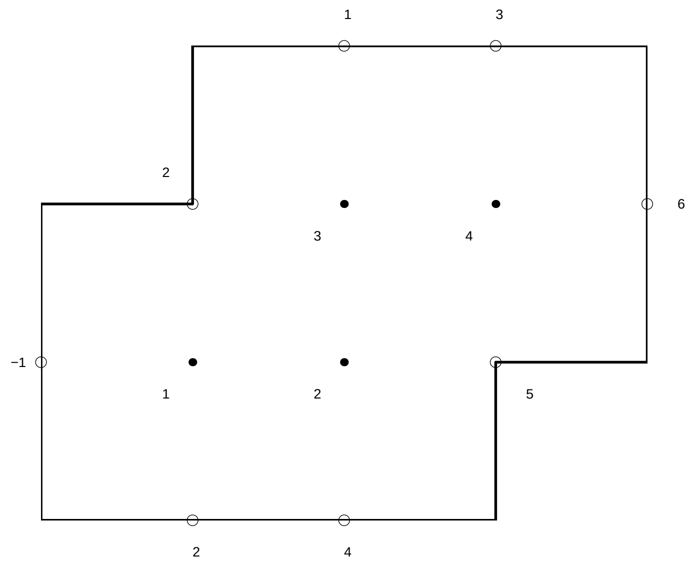
2. Minimize the Rosenbrock function with constraints $-2 \leq x \leq 0.8$ using for instance the substitution $x = -2 + 2.8 * (\sin(u))^2$.

3. Solve the Dirichlet problem in the situation of the picture and with boundary values and indexing as in the picture when the grid point distance is $h = 1$.

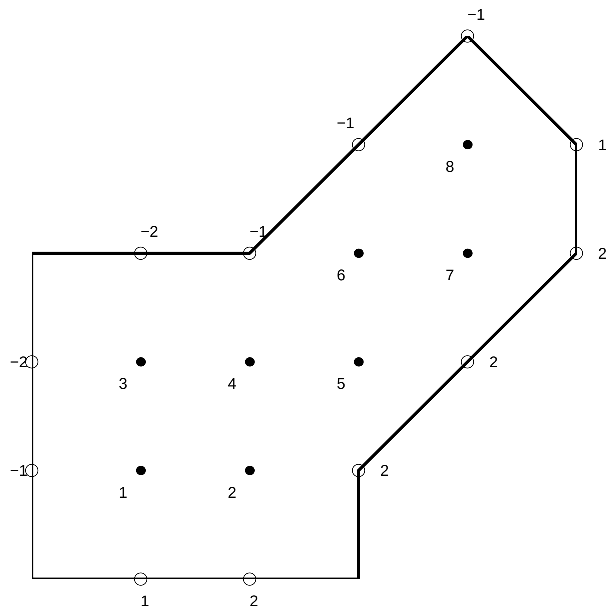


ENUMERATION OF UNKNOWNS X[1]...X[9]

4. a) Solve the Dirichlet problem in the situation of the picture, with boundary-values as in the picture, and obeying the given numbering of variables. The side length of a square is 1.



b) As a), but in the situation of the picture below.



5. For $n = 1, 2, \ldots$ let $S_n = \sum_{k=1}^{n} \sin(k\theta)/k$.

   (a) Show by experiments that $S_n \to (\pi - \theta)/2$ on $[0, \pi]$ when $n \to \infty$.

   (b) It was conjectured by L. Féjer in 1910 that $S_n > 0$ for all $\theta \in (0, \pi)$. Verify this statement experimentally.

   (c) Study also whether

$$\frac{d}{d\theta} \sum_{k=1}^{n} \frac{\sin(k\theta)}{k \sin(\theta/2)} < 0 .$$